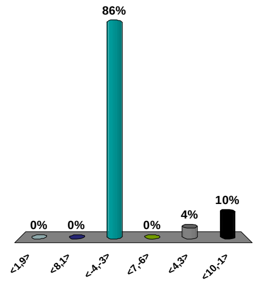
Find a vector, $\vec{a}$ with representation $\overline{AB}$, if A(7,1) and B(3,-2).

1. <1,9>
2. <8,1>
3. <-4,-3>
4. <7,-6>
5. <4,3>
6. <10,-1>

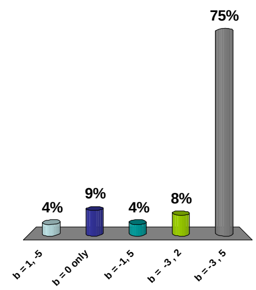


Solution:

If A(7,1) and B(3,-2), then $\overline{AB}$ = <3-7,-2-1> = <-4,-3>

For what values of b will the vectors <b,-2,3> and <b,b,-5> be perpendicular?

1. b = 1, -5
2. b = 0 only
3. b = -1, 5
4. b = -3 , 2
5. b = -3 , 5



Sln:

$$< b, -2, 3 > \cdot < b, b, -5 >= 0$$

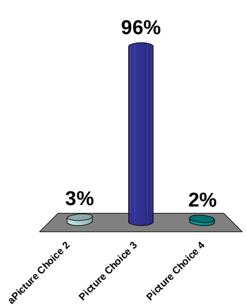
$$b^2 - 2b - 15 = 0$$

$$(b-5)(b+3) = 0$$

$$b = 5, -3$$

Which of the following is meaningful?

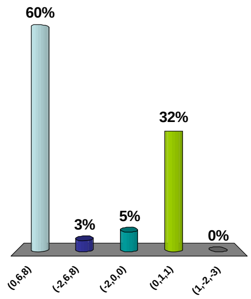
1. $(2\vec{i} \cdot 3\vec{j}) \times 4\vec{k}$
2. $(2\vec{i} \times 3\vec{j}) \cdot 4\vec{k}$
3. $(2\vec{i} \cdot 3\vec{j}) \cdot 4\vec{k}$



1. $(2\vec{i} \cdot 3\vec{j}) \times 4\vec{k}$ scalar crossed with a vector – not possible
2. $(2\vec{i} \times 3\vec{j}) \cdot 4\vec{k}$ Vector dotted with a vector 
3. $(2\vec{i} \cdot 3\vec{j}) \cdot 4\vec{k}$ scalar dotted with a vector – not possible

Where does the line x=1-2t y=-2+6t z=-3+8t intersect the yz plane?

1. (0,6,8)
2. (-2,6,8)
3. (-2,0,0)
4. (0,1,1)
5. (1,-2,-3)



Sln: x=1-2t y=-2+6t z=-3+8t

The yz plane contains all points of the form (0,y,z)

Let x = 0 then 0=1-2t and t=1/2

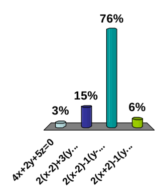
y=-2+6(1/2) = -2 +3 =1

z=-3+8(1/2) = -3 +4 =1

(x,y,z)=(0,1,1)

Find the equation of the plane through the point, $P_1(2,3,1)$ and perpendicular to $\vec{v} = <2,-1,4>$.

1. 4x+2y+5z=0
2. 2(x-2)+3(y+1)+1(z-4)=0
3. 2(x-2)-1(y-3)+4(z-1)=0
4. 2(x+2)-1(y+3)+4(z+1)=0



Sln:

Scalar equation of a plane through $P_1(x_1, y_1, z_1)$ *perpendicular to* $\vec{v} = <a,b,c>$

$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$

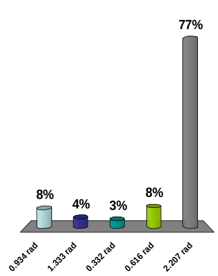
So, for $P_1(2,3,1)$ parallel to $\vec{v} = <2,-1,4>$

2(x-2)-1(y-3)+4(z-1)=0

Find the angle between the two planes:

x-2y+z=0 2x+3y-2z=0

1. 0.934 rad
2. 1.333 rad
3. 0.332 rad
4. 0.616 rad
5. 2.207 rad



Sln: x-2y+z=0 2x+3y-2z=0

The normal vectors are n_1 = <1,-2,1> and n_2 = <2,3,-2>.

The angle between the normal vectors can be found:

$$\cos\theta = \frac{n_1 \cdot n_2}{|n_1||n_2|} \qquad |n_1| = \sqrt{1^2 + (-2)^2 + (1)^2} = \sqrt{6}$$

$$|n_2| = \sqrt{2^2 + (3)^2 + (-2)^2} = \sqrt{17}$$

$$n_1 \cdot n_2 = (1)(2) + (-2)(3) + (1)(-2) = -6$$

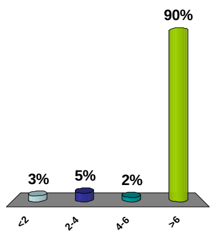
$$\cos\theta = \frac{-6}{\sqrt{6}\sqrt{17}}$$

$$\cos\theta = -0.594$$

$$\theta = 2.207 \ \ rad = 126.5^o$$

Which of the following functions require logarithmic differentiation to compute $f\,'(x)$?

1. $f(x) = \ln(\sin(x+4))$
2. $f(x) = \sin(x^2)\sqrt{x-1}$
3. $f(x) = \ln(\cos(x))$
4. $f(x) = (\cos(x+2))^{\sqrt{5+x}}$



Sln:

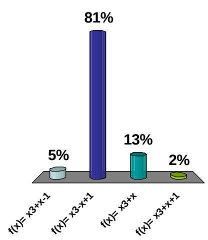
Only $f(x) = (\cos(x+2))^{\sqrt{5+x}}$

has a function as both the base and the exponent.

So, it requires logarithmic differentiation.

Which of the functions does not have an inverse?

1. f(x)= x^3+x-1
2. f(x)= x^3-x+1
3. f(x)= x^3+x
4. f(x)= x^3+x+1



Sln:

f(x)= x^3+x-1 *f*(x)= x^3-x+1 *f*(x)= x^3+x *f*(x)= x^3+x+1

Method 1:
Graph all functions and notice that only f(x)= x^3-x+1 is not 1-1. Thus, has no inverse.

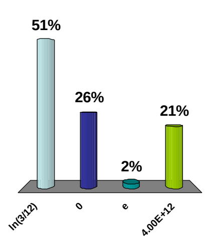
Method 2:
Calculate the derivative of each function.

f '(x)= $3x^2$ +1 *f* ' (x)= $3x^2$ -1 *f* ' (x)= $3x^2$ +1 *f* '(x)= $3x^2$ +1

Only *f* '(x) = $3x^2$-1 changes sign, indicating that the corresponding f(x) changes from increasing to decreasing. Thus, it is not 1-1 and has no inverse.

Evaluate the inverse of $f(x) = 4e^{3x}$ at x = 4.

1. ln(3/12)
2. 0
3. e
4. 4e12



Sln:

$f(x) = 4e^{3x} \quad y = 4e^{3x}$

Solve for x:

$$\frac{y}{4} = e^{3x}$$

$$\ln\left(\frac{y}{4}\right) = \ln(e^{3x})$$

$$\ln\left(\frac{y}{4}\right) = 3x$$

$$x = \frac{\ln\left(\frac{y}{4}\right)}{3} = f^{-1}(y)$$

Interchange x and y:

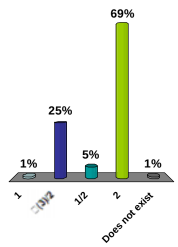
$$y = \frac{\ln\left(\frac{x}{4}\right)}{3} = f^{-1}(x)$$

Evaluate at $x = 4$:

$$y = \frac{\ln\left(\frac{4}{4}\right)}{3} = \frac{\ln(1)}{3} = 0$$

Evaluate exactly $\sec(\cos^{-1}(1/2))$.

1. 1
2. √(3)/2
3. 1/2
4. 2
5. Does not exist



Sln:

$\sec(\cos^{-1}(1/2))$

$\theta = \cos^{-1}(1/2) \Leftrightarrow \cos(\theta) = 1/2$

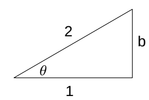
Since $y = \cos^{-1}(x)$ *is only defined for* $[0, \pi]$, $\theta = \pi/3$

$\sec(\cos^{-1}(1/2)) = \sec(\pi/3) = \frac{1}{\cos(\pi/3)} = \frac{1}{\frac{1}{2}} = 2$

–or–

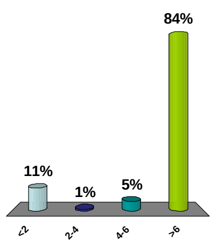
use a right triangle: $2^2 = 1^2 + b^2 \Rightarrow b = \sqrt{4-1} = \sqrt{3}$

So, $\sec(\theta) = \frac{hyp}{adj} = \frac{2}{1} = 2$



Evaluate $\int \frac{2}{x^2+9}dx$.

1. $2\ln|x^2+9| + C$
2. $-2(x^2+9)^{-2} + C$
3. $2\tan^{-1}(x) + C$
4. $\frac{2}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$

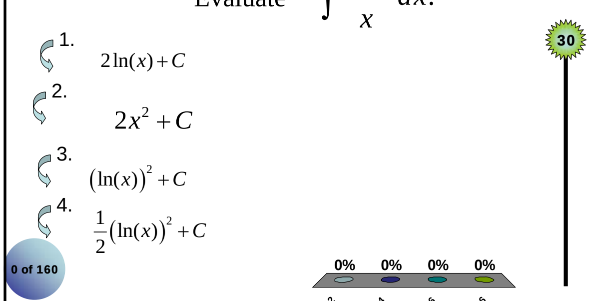


Sln:

$$\int \frac{2}{x^2+9}dx = 2\int \frac{1}{x^2+3^2}dx$$
$$= 2\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C = \frac{2}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$$

Evaluate $\int \frac{\ln x}{x}dx$.

1. $2\ln(x) + C$
2. $2x^2 + C$
3. $(\ln(x))^2 + C$
4. $\frac{1}{2}(\ln(x))^2 + C$



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

Sln:

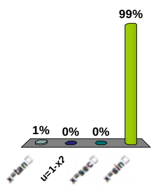
$$\int \frac{\ln x}{x}dx = \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}(\ln x)^2 + C$$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

Which substitution should be used to evaluate $\int_{-1}^{1}\sqrt{1-x^2}\,dx$?

1. x=tanθ
2. u=1-x^2
3. x=secθ
4. x=sinθ



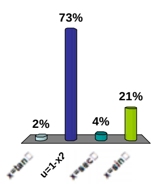
Soln:

$$\int_{-1}^{1}\sqrt{1-x^2}\,dx$$

$x=\sin\theta$ is the substitution that eliminates the radial.

Which substitution should be used to evaluate $\int_{-1}^{1}x\sqrt{1-x^2}\,dx$?

1. x=tanθ
2. u=1-x^2
3. x=secθ
4. x=sinθ



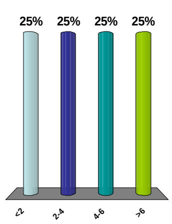
Soln:

$$\int_{-1}^{1}x\sqrt{1-x^2}\,dx$$

$u=1-x^2$ is the substitution that is easiest, but $x=\sin\theta$ would work also.

Which of the following is the form of the decomposition of $f(x)=\dfrac{x^2+5x+6}{x^3+2x^2+x}$?

1. $=\dfrac{A}{x}+\dfrac{Bx+C}{x^2+2x+1}$
2. $=\dfrac{A}{x}+\dfrac{B}{x+1}+\dfrac{C}{(x+1)^2}$
3. $=\dfrac{A}{x+2}+\dfrac{B}{x+3}$
4. $=\dfrac{A}{x}+\dfrac{B}{x+1}+\dfrac{C}{x+1}$

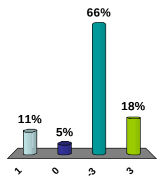


Sln:

$$f(x)=\frac{x^2+5x+6}{x^3+2x^2+x}=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{(x+1)^2}$$

Find the value of the coefficient above the (x+3) term when integrating $\int \frac{x+x^2}{x^2+4x+3}dx$?

1. 1
2. 0
3. -3
4. 3



Sln:

Long division

$$\int \frac{x+x^2}{x^2+4x+3}dx = \int 1+\frac{-3x-3}{(x+1)(x+3)}dx = \int 1+\frac{A}{x+1}+\frac{B}{x+3}dx$$

$-3x-3 = A(x+3)+B(x+1)$

$let\ x=-3$

$(-3)(-3)-3 = B(-3+1)$

$6 = B(-2)$

$B=-3$

Which table entry could be used to evaluate $\int \frac{x\sqrt{9x^2+4}}{x^3}dx$?

1. 42
2. 32
3. 33
4. 24
5. 23

Sln:

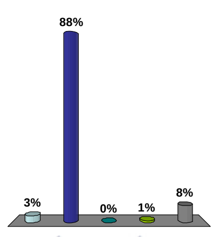
$$\int \frac{x\sqrt{9x^2+4}}{x^3}dx \quad let\ u=3x,\ du=3dx,\ \frac{1}{3}du=dx$$

$$\int \frac{\sqrt{(3x)^2+2^2}}{x^2}dx = \int \frac{\sqrt{u^2+2^2}}{\left(\frac{u}{3}\right)^2}\frac{1}{3}du = \frac{9}{3}\int \frac{\sqrt{u^2+2^2}}{u^2}du$$

$$=\frac{9}{3}\int \frac{\sqrt{2^2+u^2}}{u^2}du \quad so\ use\ table\ entry\ \#24$$

Find $\lim_{x\to\infty} \frac{e^{5x-2}}{3x^2+6}$

1. 1
2. ∞
3. e
4. - ∞
5. 0

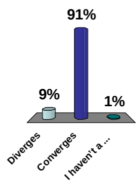


Sln:

$$\lim_{x\to\infty}\frac{e^{5x-2}}{3x^2+6} \underset{\frac{\infty}{\infty}}{\overset{LH}{=}} \lim_{x\to\infty}\frac{5e^{5x-2}}{6x} \underset{\frac{\infty}{\infty}}{\overset{LH}{=}} \lim_{x\to\infty}\frac{25e^{5x-2}}{6} = \infty$$

Determine whether the **sequence** converges or diverges $\left\{\dfrac{n+1}{3n-4}\right\}_{n=1}^{\infty}$

1. Diverges
2. Converges
3. I haven't a clue!



Soln:

$$\left\{\frac{n+1}{3n-4}\right\}_{n=1}^{\infty}$$

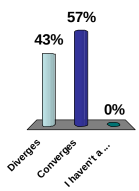
$$a_n = \frac{n+1}{3n-4}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{n+1}{3n-4} = \frac{1}{3}$$

$$\therefore \textit{The sequence converges to } \frac{1}{3}.$$

Determine whether the **series** converges or diverges $\displaystyle\sum_{n=1}^{\infty}\frac{n+1}{3n-4}$

1. Diverges
2. Converges
3. I haven't a clue!



Soln:

$$\sum_{n=1}^{\infty}\frac{n+1}{3n-4}$$

$$a_n = \frac{n+1}{3n-4}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{n+1}{3n-4} = \frac{1}{3}$$

$$\therefore \textit{The series is divergent by the test for divergence.}$$

Determine whether the **series** converges or diverges $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n 3}{2^n}$. If convergent, determine the sum.

1. Diverges
2. Converges to -3
3. Converges to -1
4. Converges to 2

Soln:

$$\sum_{n=0}^{\infty}\frac{(-1)^n 3}{2^n} = \sum_{n=0}^{\infty} 3\left(\frac{-1}{2}\right)^n$$

$$a = 3 \quad r = \frac{-1}{2}$$

$$so\ |r| < 1$$

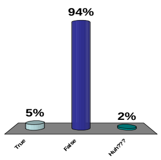
$$\therefore \textit{convergent Geometric series}$$

$$\sum_{n=0}^{\infty}\frac{(-1)^n 3}{2^n} = \frac{a}{1-r} = \frac{3}{1-\frac{-1}{2}} = \frac{3}{\frac{3}{2}} = 2$$

True or False: Assume both a_n and b_n are positive.

If $\sum_{n=1}^{\infty} b_n$ is convergent and $\lim_{n\to\infty}\frac{a_n}{b_n}=0,$ then $\sum_{n=1}^{\infty} a_n$ is divergent.

1. True
2. False
3. Huh???



Soln:

False!

The limit comparison test states:

If $\lim_{n\to\infty}\frac{a_n}{b_n}=L,\ with\ 0<L<\infty,$ then either both $\sum_{n=1}^{\infty} a_n\ and\ \sum_{n=1}^{\infty} b_n$ converge or both diverge.

Since, we have $\lim_{n\to\infty}\frac{a_n}{b_n}=0$ then the limit comparison test is inconclusive.

For example, consider the series

$$\sum_{n=1}^{\infty} a_n=\sum_{n=1}^{\infty}\frac{1}{n^2}\ and\ \sum_{n=1}^{\infty} b_n=\sum_{n=1}^{\infty}\frac{1}{n} \qquad \sum_{n=1}^{\infty} a_n=\sum_{n=1}^{\infty}\frac{1}{n^3}\ and\ \sum_{n=1}^{\infty} b_n=\sum_{n=1}^{\infty}\frac{1}{n^2}$$

$$\lim_{n\to\infty}\frac{a_n}{b_n}=\lim_{n\to\infty}\frac{\frac{1}{n^2}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{n}{n^2}=0 \qquad \lim_{n\to\infty}\frac{a_n}{b_n}=\lim_{n\to\infty}\frac{\frac{1}{n^3}}{\frac{1}{n^2}}=\lim_{n\to\infty}\frac{n^2}{n^3}=0$$

The series $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt[3]{n}}$ is:

1. Absolutely Convergent
2. Conditionally Convergent
3. Divergent

Soln:

$\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt[3]{n}}$ (The Ratio and Root Tests are inconclusive)

Alternating series test:

$$b_n=\frac{1}{\sqrt[3]{n}} \qquad f(x)=\frac{1}{\sqrt[3]{x}}=x^{-1/3}$$

$$i)decreasing\ \ f'(x)=\frac{-1}{3}x^{-4/3}<0 \quad ii)\lim_{n\to\infty}b_n=\lim_{n\to\infty}\frac{1}{\sqrt[3]{n}}=0$$

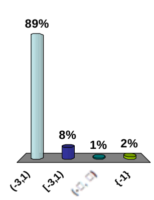
$\therefore Convergent$

$$\sum_{n=1}^{\infty}|a_n|=\sum_{n=1}^{\infty}\frac{1}{n^{1/3}} \qquad p-series\ with\ p<1 \quad \therefore Divergent$$

$The\ series\ \sum_{n=1}^{\infty}\frac{(-1)^n}{n^{1/3}}\ \ is\ conditionally\ convergent.$

Find the interval of convergence for $\sum_{n=1}^{\infty}\frac{(-1)^n(x+1)^n}{2^n}$

1. (-3,1)
2. [-3,1)
3. (-∞, ∞)
4. {-1}



Soln:

$$\sum_{n=1}^{\infty}\frac{(-1)^n(x+1)^n}{2^n}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=\lim_{n\to\infty}\left|\frac{(x+1)^{n+1}}{2^{n+1}}\cdot\frac{2^n}{(x+1)^n}\right|=\left|\frac{(x+1)}{2}\right|$$

$$Converges\ if\ \left|\frac{(x+1)}{2}\right|<1\Rightarrow|x+1|<2\Rightarrow(-3,1)$$

$Testing\ Endpoits:$

$x=-3$

$$\sum_{n=1}^{\infty}\frac{(-1)^n(-3+1)^n}{2^n}=\sum_{n=1}^{\infty}\frac{(-1)^n(-2)^n}{2^n}=\sum_{n=1}^{\infty}\frac{2^n}{2^n}=\sum_{n=1}^{\infty}1\ (Diverges)$$

$x=1$

$$\sum_{n=1}^{\infty}\frac{(-1)^n(1+1)^n}{2^n}=\sum_{n=1}^{\infty}\frac{(-1)^n(2)^n}{2^n}=\sum_{n=1}^{\infty}(-1)^n\ \ (Diverges)$$

$\therefore I=(-3,1)$

Find a power series representation for $f(x)=\frac{x}{1-5x}$

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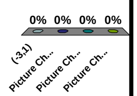
1. $\sum_{n=0}^{\infty}\frac{5^n x^{n+1}}{n+1}$
2. $\sum_{n=0}^{\infty}5^n x^n$
3. $\sum_{n=0}^{\infty}(5)^n x^{n+1}$
4. $\sum_{n=0}^{\infty}\frac{(5x)^{n+2}}{n+2}$



0 of 5

1	2	3	4	5															

Soln:

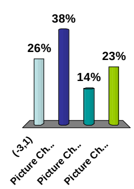
$$Know: \frac{1}{1-x}=\sum_{n=0}^{\infty}x^n$$

$$\frac{1}{1-5x}=\sum_{n=0}^{\infty}(5x)^n=\sum_{n=0}^{\infty}5^n x^n$$

$$\frac{x}{1-5x}=x\sum_{n=0}^{\infty}5^n x^n=\sum_{n=0}^{\infty}5^n x^{n+1}$$

Find a power series representation for $\int\frac{x}{1-5x}dx.$

1. $\sum_{n=0}^{\infty}\frac{5^n x^{n+1}}{n+1}+C$
2. $\sum_{n=0}^{\infty}\frac{5^n x^{n+2}}{n+2}+C$
3. $\sum_{n=0}^{\infty}\frac{(-5)^n x^{n+2}}{n+2}+C$
4. $\sum_{n=0}^{\infty}\frac{(5x)^{n+2}}{n+2}+C$



Soln:

$$Know: \frac{1}{1-x}=\sum_{n=0}^{\infty}x^n$$

$$\frac{1}{1-5x}=\sum_{n=0}^{\infty}(5x)^n=\sum_{n=0}^{\infty}5^n x^n$$

$$\frac{x}{1-5x}=x\sum_{n=0}^{\infty}5^n x^n=\sum_{n=0}^{\infty}5^n x^{n+1}$$

$$\int\frac{x}{1-5x}dx=\int\sum_{n=0}^{\infty}5^n x^{n+1}dx=\sum_{n=0}^{\infty}\frac{5^n x^{n+2}}{n+2}+C$$

Find the domain and range of $f(x,y)=-e^{\sqrt{2x+y}}$

1. D:{(x,y)|y=-2x} R:{z|z≥0}
2. D:{(x,y)|ℝ²} R:{z|z>0}
3. D:{(x,y)|ℝ²} R:{z|ℝ}
4. D:{(x,y)|y≥-2x} R:{z|z<0}

Soln:

$f(x,y)=-e^{\sqrt{2x+y}}$

$need\ 2x+y\geq 0$

$D:\{(x,y)\mid y\geq -2x\}$

$Range$

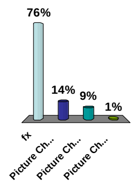
$Since\ the\ range\ of\ \ f(x)=e^x\ \ is\ (0,\infty)$

$Then\ the\ range\ of\ \ f(x)=-e^x\ \ is\ (-\infty,0)$

$Thus, R:\{z\mid z<0\}$

For $f(x,y) = (2x-3y)^3$, is f_x *or* f_y larger at the point (1,1)?

1. f_x
2. f_y
3. $f_x = f_y$
4. $say\ what?$



Soln:

$f(x,y) = (2x-3y)^3$

$f_x(x,y) = 3(2x-3y)^2(2)$

$f_x(1,1) = 3(2(1)-3(1))^2(2) = 6$

$f_y(x,y) = 3(2x-3y)^2(-3)$

$f_y(1,1) = 3(2(1)-3(1))^2(-3) = -9$

Thus, $f_x > f_y$.