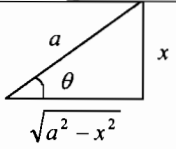
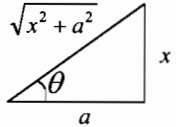
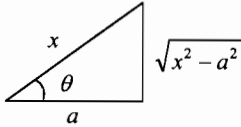


Math 1297- Exam 2 – Formula Sheet

Inverse Trig Derivatives	Hyperbolic Derivatives	Inverse Hyperbolic Derivatives	Hyperbolic Functions
$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\sinh x) = \cosh x$	$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$	$\sinh x = \frac{e^x - e^{-x}}{2}$
$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\cosh x) = \sinh x$	$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$	$\cosh x = \frac{e^x + e^{-x}}{2}$
$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$	$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$	$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$	$\sinh^{-1} x = \ln(x + \sqrt{x^2+1})$
$\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$	$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$	$\frac{d}{dx}(\operatorname{csch}^{-1} x) = \frac{-1}{ x \sqrt{x^2+1}}$	$\cosh^{-1} x = \ln(x + \sqrt{x^2-1})$
$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$	$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$	$\frac{d}{dx}(\operatorname{sech}^{-1} x) = \frac{-1}{x\sqrt{1-x^2}}$	$\tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$
$\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$	$\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$	$\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}$	

Expression:	Substitution	Identity	Triangle
$\sqrt{a^2 - x^2}$	$x = a \sin \theta, \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$	$\cos^2 \theta = 1 - \sin^2 \theta$	
$\sqrt{x^2 + a^2}$	$x = a \tan \theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$	$\sec^2 \theta = 1 + \tan^2 \theta$	
$\sqrt{x^2 - a^2}$	$x = a \sec \theta, \quad 0 \leq \theta < \frac{\pi}{2} \text{ or } \pi \leq \theta < \frac{3\pi}{2}$	$\tan^2 \theta = \sec^2 \theta - 1$	

Useful Trig Identities

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin x \cos x = \frac{1}{2} \sin 2x$$

8 Table of Integrals

Forms Involving $a + bu$

$$1. \int \frac{1}{a+bu} du = \frac{1}{b} \ln |a+bu| + c$$

$$2. \int \frac{u}{a+bu} du = \frac{1}{b^2} (a+bu - a \ln |a+bu|) + c$$

$$3. \int \frac{u^2}{a+bu} du = \frac{1}{2b^3} [(a+bu)^2 - 4a(a+bu) + 2a^2 \ln |a+bu|] + c$$

$$4. \int \frac{1}{u(a+bu)} du = \frac{1}{a} \ln \left| \frac{u}{a+bu} \right| + c$$

$$5. \int \frac{1}{u^2(a+bu)} du = \frac{b}{a^2} \ln \left| \frac{a+bu}{u} \right| - \frac{1}{au} + c$$

Forms Involving $(a+bu)^2$

$$6. \int \frac{1}{(a+bu)^2} du = \frac{-1}{b(a+bu)} + c$$

$$7. \int \frac{u}{(a+bu)^2} du = \frac{1}{b^2} \left(\frac{a}{a+bu} + \ln |a+bu| \right) + c$$

$$8. \int \frac{u^2}{(a+bu)^2} du = \frac{1}{b^3} \left(a+bu - \frac{a^2}{a+bu} - 2a \ln |a+bu| \right) + c$$

$$9. \int \frac{1}{u(a+bu)^2} du = \frac{1}{a(a+bu)} + \frac{1}{a^2} \ln \left| \frac{u}{a+bu} \right| + c$$

$$10. \int \frac{1}{u^2(a+bu)^2} du = \frac{2b}{a^3} \ln \left| \frac{a+bu}{u} \right| - \frac{a+2bu}{a^2 u(a+bu)} + c$$

Forms Involving $\sqrt{a+bu}$

$$11. \int u\sqrt{a+bu} du = \frac{2}{15b^2} (3bu-2a)(a+bu)^{3/2} + c$$

$$12. \int u^2\sqrt{a+bu} du = \frac{2}{105b^3} (15b^2u^2 - 12abu + 8a^2)(a+bu)^{3/2} + c$$

$$13. \int u^n\sqrt{a+bu} du = \frac{2}{b(2n+3)} u^n(a+bu)^{3/2} - \frac{2na}{b(2n+3)} \int u^{n-1}\sqrt{a+bu} du$$

$$14. \int \frac{\sqrt{a+bu}}{u} du = 2\sqrt{a+bu} + a \int \frac{1}{u\sqrt{a+bu}} du$$

$$15. \int \frac{\sqrt{a+bu}}{u^n} du = \frac{-1}{a(n-1)} \frac{(a+bu)^{3/2}}{u^{n-1}} - \frac{(2n-5)b}{2a(n-1)} \int \frac{\sqrt{a+bu}}{u^{n-1}} du, n \neq 1$$

$$16a. \int \frac{1}{u\sqrt{a+bu}} du = \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a+bu} - \sqrt{a}}{\sqrt{a+bu} + \sqrt{a}} \right| + c, a > 0$$

$$16b. \int \frac{1}{u\sqrt{a+bu}} du = \frac{2}{\sqrt{-a}} \tan^{-1} \sqrt{\frac{a+bu}{-a}} + c, a < 0$$

$$17. \int \frac{1}{u^n\sqrt{a+bu}} du = \frac{-1}{a(n-1)} \frac{\sqrt{a+bu}}{u^{n-1}} - \frac{(2n-3)b}{2a(n-1)} \int \frac{1}{u^{n-1}\sqrt{a+bu}} du, n \neq 1$$

$$18. \int \frac{u}{\sqrt{a+bu}} du = \frac{2}{3b^2} (bu-2a)\sqrt{a+bu} + c$$

$$19. \int \frac{u^2}{\sqrt{a+bu}} du = \frac{2}{15b^3} (3b^2u^2 - 4abu + 8a^2)\sqrt{a+bu} + c$$

$$20. \int \frac{u^n}{\sqrt{a+bu}} du = \frac{2}{(2n+1)b} u^n\sqrt{a+bu} - \frac{2na}{(2n+1)b} \int \frac{u^{n-1}}{\sqrt{a+bu}} du$$

Forms Involving $\sqrt{a^2+u^2}$, $a > 0$

$$21. \int \sqrt{a^2+u^2} du = \frac{1}{2} u\sqrt{a^2+u^2} + \frac{1}{2} a^2 \ln |u + \sqrt{a^2+u^2}| + c$$

$$22. \int u^2\sqrt{a^2+u^2} du = \frac{1}{8} u(a^2+2u^2)\sqrt{a^2+u^2} - \frac{1}{8} a^4 \ln |u + \sqrt{a^2+u^2}| + c$$

$$23. \int \frac{\sqrt{a^2+u^2}}{u} du = \sqrt{a^2+u^2} - a \ln \left| \frac{a + \sqrt{a^2+u^2}}{u} \right| + c$$

$$24. \int \frac{\sqrt{a^2+u^2}}{u^2} du = \ln |u + \sqrt{a^2+u^2}| - \frac{\sqrt{a^2+u^2}}{u} + c$$

$$25. \int \frac{1}{\sqrt{a^2+u^2}} du = \ln |u + \sqrt{a^2+u^2}| + c$$

$$26. \int \frac{u^2}{\sqrt{a^2+u^2}} du = \frac{1}{2} u\sqrt{a^2+u^2} - \frac{1}{2} a^2 \ln |u + \sqrt{a^2+u^2}| + c$$

$$27. \int \frac{1}{u\sqrt{a^2+u^2}} du = \frac{1}{a} \ln \left| \frac{u}{a + \sqrt{a^2+u^2}} \right| + c$$

$$28. \int \frac{1}{u^2\sqrt{a^2+u^2}} du = -\frac{\sqrt{a^2+u^2}}{a^2u} + c$$

Forms Involving $\sqrt{a^2-u^2}$, $a > 0$

$$29. \int \sqrt{a^2-u^2} du = \frac{1}{2} u\sqrt{a^2-u^2} + \frac{1}{2} a^2 \sin^{-1} \frac{u}{a} + c$$

$$30. \int u^2\sqrt{a^2-u^2} du = \frac{1}{8} u(2u^2-a^2)\sqrt{a^2-u^2} + \frac{1}{8} a^4 \sin^{-1} \frac{u}{a} + c$$

$$31. \int \frac{\sqrt{a^2-u^2}}{u} du = \sqrt{a^2-u^2} - a \ln \left| \frac{a + \sqrt{a^2-u^2}}{u} \right| + c$$

$$32. \int \frac{\sqrt{a^2-u^2}}{u^2} du = -\frac{\sqrt{a^2-u^2}}{u} - \sin^{-1} \frac{u}{a} + c$$

$$33. \int \frac{1}{\sqrt{a^2 - u^2}} du = \sin^{-1} \frac{u}{a} + c$$

$$34. \int \frac{1}{u\sqrt{a^2 - u^2}} du = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + c$$

$$35. \int \frac{u^2}{\sqrt{a^2 - u^2}} du = -\frac{1}{2}u\sqrt{a^2 - u^2} + \frac{1}{2}a^2 \sin^{-1} \frac{u}{a} + c$$

$$36. \int \frac{1}{u^2\sqrt{a^2 - u^2}} du = -\frac{\sqrt{a^2 - u^2}}{a^2 u} + c$$

Forms Involving $\sqrt{u^2 - a^2}$, $a > 0$

$$37. \int \sqrt{u^2 - a^2} du = \frac{1}{2}u\sqrt{u^2 - a^2} - \frac{1}{2}a^2 \ln |u + \sqrt{u^2 - a^2}| + c$$

$$38. \int u^2 \sqrt{u^2 - a^2} du = \frac{1}{8}u(2u^2 - a^2)\sqrt{u^2 - a^2} - \frac{1}{8}a^4 \ln |u + \sqrt{u^2 - a^2}| + c$$

$$39. \int \frac{\sqrt{u^2 - a^2}}{u} du = \sqrt{u^2 - a^2} - a \sec^{-1} \frac{|u|}{a} + c$$

$$40. \int \frac{\sqrt{u^2 - a^2}}{u^2} du = \ln |u + \sqrt{u^2 - a^2}| - \frac{\sqrt{u^2 - a^2}}{u} + c$$

$$41. \int \frac{1}{\sqrt{u^2 - a^2}} du = \ln |u + \sqrt{u^2 - a^2}| + c$$

$$42. \int \frac{u^2}{\sqrt{u^2 - a^2}} du = \frac{1}{2}u\sqrt{u^2 - a^2} + \frac{1}{2}a^2 \ln |u + \sqrt{u^2 - a^2}| + c$$

$$43. \int \frac{1}{u\sqrt{u^2 - a^2}} du = \frac{1}{a} \sec^{-1} \frac{|u|}{a} + c$$

$$44. \int \frac{1}{u^2\sqrt{u^2 - a^2}} du = \frac{\sqrt{u^2 - a^2}}{a^2 u} + c$$

Forms Involving $\sqrt{2au - u^2}$

$$45. \int \sqrt{2au - u^2} du = \frac{1}{2}(u - a)\sqrt{2au - u^2} + \frac{1}{2}a^2 \cos^{-1} \left(\frac{a - u}{a} \right) + c$$

$$46. \int u\sqrt{2au - u^2} du = \frac{1}{6}(2u^2 - au - 3a^2)\sqrt{2au - u^2} + \frac{1}{2}a^3 \cos^{-1} \left(\frac{a - u}{a} \right) + c$$

$$47. \int \frac{\sqrt{2au - u^2}}{u} du = \sqrt{2au - u^2} + a \cos^{-1} \left(\frac{a - u}{a} \right) + c$$

$$48. \int \frac{\sqrt{2au - u^2}}{u^2} du = -\frac{2\sqrt{2au - u^2}}{u} - \cos^{-1} \left(\frac{a - u}{a} \right) + c$$

$$49. \int \frac{1}{\sqrt{2au - u^2}} du = \cos^{-1} \left(\frac{a - u}{a} \right) + c$$

$$50. \int \frac{u}{\sqrt{2au - u^2}} du = -\sqrt{2au - u^2} + a \cos^{-1} \left(\frac{a - u}{a} \right) + c$$

$$51. \int \frac{u^2}{\sqrt{2au - u^2}} du = -\frac{1}{2}(u + 3a)\sqrt{2au - u^2} + \frac{3}{2}a^2 \cos^{-1} \left(\frac{a - u}{a} \right) + c$$

$$52. \int \frac{1}{u\sqrt{2au - u^2}} du = -\frac{\sqrt{2au - u^2}}{au} + c$$

Forms Involving $\sin u$ OR $\cos u$

$$53. \int \sin u du = -\cos u + c$$

$$54. \int \cos u du = \sin u + c$$

$$55. \int \sin^2 u du = \frac{1}{2}u - \frac{1}{2}\sin u \cos u + c$$

$$56. \int \cos^2 u du = \frac{1}{2}u + \frac{1}{2}\sin u \cos u + c$$

$$57. \int \sin^3 u du = -\frac{2}{3}\cos u - \frac{1}{3}\sin^2 u \cos u + c$$

$$58. \int \cos^3 u du = \frac{2}{3}\sin u + \frac{1}{3}\sin u \cos^2 u + c$$

$$59. \int \sin^n u du = -\frac{1}{n}\sin^{n-1} u \cos u + \frac{n-1}{n} \int \sin^{n-2} u du$$

$$60. \int \cos^n u du = \frac{1}{n}\cos^{n-1} u \sin u + \frac{n-1}{n} \int \cos^{n-2} u du$$

$$61. \int u \sin u du = \sin u - u \cos u + c$$

$$62. \int u \cos u du = \cos u + u \sin u + c$$

$$63. \int u^n \sin u du = -u^n \cos u + n \int u^{n-1} \cos u du + c$$

$$64. \int u^n \cos u du = u^n \sin u - n \int u^{n-1} \sin u du + c$$

$$65. \int \frac{1}{1 + \sin u} du = \tan u - \sec u + c$$

$$66. \int \frac{1}{1 - \sin u} du = \tan u + \sec u + c$$

$$67. \int \frac{1}{1 + \cos u} du = -\cot u + \csc u + c$$

$$68. \int \frac{1}{1 - \cos u} du = -\cot u - \csc u + c$$

$$69. \int \sin(mu) \sin(nu) du = \frac{\sin(m-n)u}{2(m-n)} - \frac{\sin(m+n)u}{2(m+n)} + c$$

$$70. \int \cos(mu) \cos(nu) du = \frac{\sin(m-n)u}{2(m-n)} + \frac{\sin(m+n)u}{2(m+n)} + c$$

$$71. \int \sin(mu) \cos(nu) du = \frac{\cos(n-m)u}{2(n-m)} - \frac{\cos(m+n)u}{2(m+n)} + c$$

$$72. \int \sin^m u \cos^n u du = -\frac{\sin^{m-1} u \cos^{n+1} u}{m+n} + \frac{m-1}{m+n} \int \sin^{m-2} u \cos^n u du$$

Forms Involving Other Trigonometric Functions

$$73. \int \tan u du = -\ln |\cos u| + c = \ln |\sec u| + c$$

$$74. \int \cot u du = \ln |\sin u| + c$$

$$75. \int \sec u du = \ln |\sec u + \tan u| + c$$

$$76. \int \csc u du = \ln |\csc u - \cot u| + c$$

$$77. \int \tan^2 u du = \tan u - u + c$$

$$78. \int \cot^2 u du = -\cot u - u + c$$

$$79. \int \sec^2 u du = \tan u + c$$

$$80. \int \csc^2 u du = -\cot u + c$$

$$81. \int \tan^3 u du = \frac{1}{2} \tan^2 u + \ln |\cos u| + c$$

$$82. \int \cot^3 u du = -\frac{1}{2} \cot^2 u - \ln |\sin u| + c$$

$$83. \int \sec^3 u du = \frac{1}{2} \sec u \tan u + \frac{1}{2} \ln |\sec u + \tan u| + c$$

$$84. \int \csc^3 u du = -\frac{1}{2} \csc u \cot u + \frac{1}{2} \ln |\csc u - \cot u| + c$$

$$85. \int \tan^n u du = \frac{1}{n-1} \tan^{n-1} u - \int \tan^{n-2} u du, n \neq 1$$

$$86. \int \cot^n u du = -\frac{1}{n-1} \cot^{n-1} u - \int \cot^{n-2} u du, n \neq 1$$

$$87. \int \sec^n u du = \frac{1}{n-1} \sec^{n-2} u \tan u + \frac{n-2}{n-1} \int \sec^{n-2} u du, n \neq 1$$

$$88. \int \csc^n u du = -\frac{1}{n-1} \csc^{n-2} u \cot u + \frac{n-2}{n-1} \int \csc^{n-2} u du, n \neq 1$$

$$89. \int \frac{1}{1 \pm \tan u} du = \frac{1}{2} u \pm \ln |\cos u \pm \sin u| + c$$

$$90. \int \frac{1}{1 \pm \cot u} du = \frac{1}{2} u \mp \ln |\sin u \pm \cos u| + c$$

$$91. \int \frac{1}{1 \pm \sec u} du = u + \cot u \mp \csc u + c$$

$$92. \int \frac{1}{1 \pm \csc u} du = u - \tan u \pm \sec u + c$$

Forms Involving Inverse Trigonometric Functions

$$93. \int \sin^{-1} u du = u \sin^{-1} u + \sqrt{1-u^2} + c$$

$$94. \int \cos^{-1} u du = u \cos^{-1} u - \sqrt{1-u^2} + c$$

$$95. \int \tan^{-1} u du = u \tan^{-1} u - \ln \sqrt{1+u^2} + c$$

$$96. \int \cot^{-1} u du = u \cot^{-1} u + \ln \sqrt{1+u^2} + c$$

$$97. \int \sec^{-1} u du = u \sec^{-1} u - \ln |u + \sqrt{u^2-1}| + c$$

$$98. \int \csc^{-1} u du = u \csc^{-1} u + \ln |u + \sqrt{u^2-1}| + c$$

$$99. \int u \sin^{-1} u du = \frac{1}{4} (2u^2-1) \sin^{-1} u + \frac{1}{4} u \sqrt{1-u^2} + c$$

$$100. \int u \cos^{-1} u du = \frac{1}{4} (2u^2-1) \cos^{-1} u - \frac{1}{4} u \sqrt{1-u^2} + c$$

Forms Involving e^u

$$101. \int e^{au} du = \frac{1}{a} e^{au} + c$$

$$102. \int u e^{au} du = \left(\frac{1}{a} u - \frac{1}{a^2} \right) e^{au} + c$$

$$103. \int u^2 e^{au} du = \left(\frac{1}{a} u^2 - \frac{2}{a^2} u + \frac{2}{a^3} \right) e^{au} + c$$

$$104. \int u^n e^{au} du = \frac{1}{a} u^n e^{au} - \frac{n}{a} \int u^{n-1} e^{au} du$$

$$105. \int e^{au} \sin bu du = \frac{1}{a^2+b^2} (a \sin bu - b \cos bu) e^{au} + c$$

$$106. \int e^{au} \cos bu du = \frac{1}{a^2+b^2} (a \cos bu + b \sin bu) e^{au} + c$$

Forms Involving $\ln u$

$$107. \int \ln u du = u \ln u - u + c$$

$$108. \int u \ln u du = \frac{1}{2} u^2 \ln u - \frac{1}{4} u^2 + c$$

$$109. \int u^n \ln u du = \frac{1}{n+1} u^{n+1} \ln u - \frac{1}{(n+1)^2} u^{n+1} + c$$

$$110. \int \frac{1}{u \ln u} du = \ln |\ln u| + c$$

$$111. \int (\ln u)^2 du = u (\ln u)^2 - 2u \ln u + 2u + c$$

$$112. \int (\ln u)^n du = u (\ln u)^n - n \int (\ln u)^{n-1} du$$