Math 1290 – Fall 2008 – Exam 1 – February 28, 2008 74.4 Name: Key

1. Find the domain and range of $f(x)=\sqrt{3-x}$.

D: $(-\infty, 3]$

R $[0,\infty)$

2. Large areas are often measured in hectres (ha) or in acres. If 1 ha = 10,000 m^2 and 1 acre = 4046.86 m^2, how many acres are one hectare?

$$\frac{1\text{ ha}}{} \cdot \frac{10{,}000\text{ m}^2}{1\text{ ha}} \cdot \frac{1\text{ acre}}{4046.86\text{ m}^2} = 2.47\text{ acres}$$

3. Find the equation of the line passing thought the points (3,2) and (5,3). Use this equation to determine the y-coordinate corresponding to x=4.

$$m = \frac{\Delta y}{\Delta x} = \frac{3-2}{5-3} = \frac{1}{2}$$

$$y = \frac{1}{2}x + b$$

$$2 = \frac{1}{2}(3) + b$$

$$2 - \frac{3}{2} = b$$

$$\frac{1}{2} = b \Rightarrow \boxed{y = \frac{1}{2}x + \frac{1}{2}}$$

$$y(4) = \frac{1}{2}(4) + \frac{1}{2}$$

$$= \boxed{2.5}$$

4. For the ~~co~~sine function graphed, determine the following:

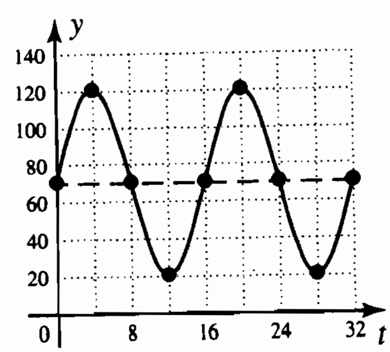
a) Minimum 20

b) Maximum 120

c) Amplitude 50

d) Period 16

e) Average 70



5. Find the inverse of $f(x) = 2x^3 + 1$.

$$y = 2x^3 + 1$$

$$y - 1 = 2x^3$$

$$\frac{y-1}{2} = x^3$$

$$\left(\frac{y-1}{2}\right)^{\frac{1}{3}} = x$$

$$f^{-1}(x) = \left(\frac{x-1}{2}\right)^{\frac{1}{3}}$$

6. Let $b_{t+1} = 0.4b_t$. *If* $b_0 = 25$, find the solution of the Discrete-Time Dynamical system and express this solution using base *e*. How long does it take for the population to reach [redacted] 5?

$$b(t) = b_0(0.4)^t = 25(0.4)^t = 25e^{\ln(0.4)t} = 25e^{-0.916t}$$

$$5 = 25e^{-0.916t}$$

$$\frac{5}{25} = e^{-0.916t}$$

$$\ln\left(\frac{5}{25}\right) = -0.916t$$

$$\boxed{t = 1.756}$$

7. A worm farm has a per capita production of 5. If 40 worms are harvested every day to be sold at the bait shop, write the Discrete-Time Dynamical System and determine the equilibrium population.

$$W_{t+1} = 5W_t - 40$$

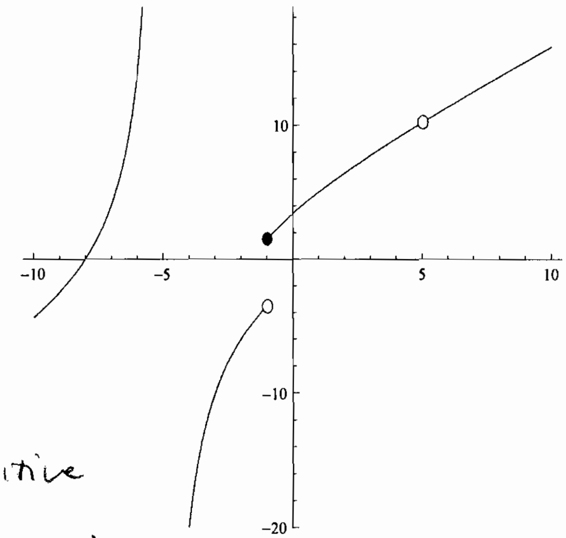
let $w^* = w_t = w_{t+1}$

$$w^* = 5w^* - 40$$

$$-4w^* = -40$$

$$\boxed{w^* = 10}$$

8. Use the graph to determine the following:



a) $\lim_{x\to -5^+} f(x) = -\infty$

b) $f(-1) = 2$

c) $\lim_{x\to 5} f(x) = 10$

d) T/F $\dfrac{f(7)-f(1)}{7-1} < 0$ False, m_{sec} is positive

e) T/F $f'(-2) > 0$ True, m_{tan} at $x=-1$ is positive

9. At which values is *f(x)* discontinuous? State the reason (in terms of limits) for the discontinuity and the type of discontinuity.

$$f(x)=\begin{cases}\dfrac{1}{x} & for\ x<3\\ x^2+1 & for\ x\ge 3\end{cases}$$

$x=0$ $\quad \lim_{x\to 0^+} f(x) \neq \lim_{x\to 0^-} f(x)$ infinite discontinuity

$x=3$ $\quad \lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} x^2+1 = 10$
and
$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{1}{x} = \frac{1}{3}$ jump discontinuity

10. Using the limit definition of the derivative find the slope of the line tangent to the curve $f(x)=x^2+2$ at x=2.

$$m_{tan} = \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x} = \lim_{\Delta x\to 0}\frac{(x+\Delta x)^2+2-(x^2+2)}{\Delta x} = \lim_{\Delta x\to 0}\frac{\cancel{x^2}+2x\Delta x+(\Delta x)^2+\cancel{2}-\cancel{x^2}-\cancel{2}}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\frac{2x\Delta x+(\Delta x)^2}{\Delta x} = \lim_{\Delta x\to 0}\frac{\cancel{\Delta x}(2x+\Delta x)}{\cancel{\Delta x}} = 2x$$

at $x=2$ $\quad m_{tan} = f'(2) = 2(2) = \boxed{4}$