Math 1290 – Spring 2008 – Exam 2 – March 31st, 2008 Name: Key

1. Find the equation of the line tangent to the curve $f(x) = x^3 \ln x$ at $x = 1$.

$f(1) = 1^3 \ln(1) = 0$ +2

$f'(x) = x^3 \frac{1}{x} + \ln x \cdot 3x^2 = x^2 + 3x^2 \ln x$ +3

$f'(1) = 1^2 + 3(1)^2 \ln(1) = 1$ +2

$y = 1x + b$

$0 = 1(1) + b \Rightarrow b = -1$ +1

$\boxed{y = 1x - 1}$ +1

2. On Mars the height in feet of a baseball thrown straight up by Walter Johnson (the fastest pitcher in Major League history) with his record velocity of 134 ft/sec can be described by $h(t) = -6.27t^2 + 134t$

a.) Determine the average velocity of the ball from t = 2 secs to t = 6 secs.

+2 $V_{avg} = \frac{h(6)-h(2)}{6-2} = \frac{578.28 - 242.92}{4} = \frac{335.36}{4} = \boxed{83.84 \text{ ft/sec}}$

b.) Determine the instantaneous velocity of the ball at t = 4.5 secs.

+3 $v(t) = h'(t) = -12.54t + 134$

$h'(4.5) = -12.54(4.5) + 134 = \boxed{77.57 \text{ ft/sec}}$

c.) Determine the time when the ball reaches it maximum height.

$h'(t) = 0$

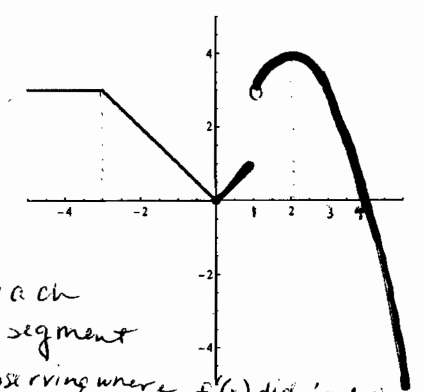
+3 $-12.54t + 134 = 0$

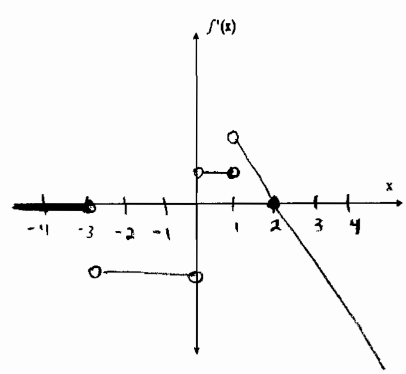
$t = \frac{-134}{-12.54} = \boxed{10.69 \text{ sec}}$

d.) What is the maximal height?

+2 $h(10.69) = -6.27(10.69)^2 + 134(10.69) = \boxed{715.95 \text{ ft}}$

3. Sketch a graph of the derivative using the graph of $f(x)$ provided below.





+2 for each line segment

+2 for observing where f'(x) did n't exist

4. An environmental study indicates that the average daily level P of a certain pollutant in the air in parts per million can be modeled by the equation $P(n) = 0.25\sqrt{0.5n^2 + 5n + 25}$ where n is the number of residents of the community in thousands. Find the rate at which the level of pollutant is increasing when the population is 12,000.

$P'(n) = 0.25 \frac{1}{2} (0.5n^2 + 5n + 25)^{-\frac{1}{2}} (n + 5)$ +5

$P'(12) = 0.25 \frac{1}{2} (0.5(12)^2 + 5(12) + 25)^{-\frac{1}{2}} (12 + 5)$ +4

$P'(12) = 0.169 \frac{ppm}{resident}$ +1

5. If a population has a per capita production of 4-2b_t, use the Slope Criterion to determine the stability of the equilibrium point(s).

+4 $b_{t+1} = (4-2b_t)b_t \Rightarrow f(b_t) = (4-2b_t)b_t = 4b_t - 2b_t^2$

$b^* = 4b^* - 2(b^*)^2$

$0 = 3b^* - 2(b^*)^2$

$0 = b^*(3 - 2b^*)$

$b^* = 0 \quad 3 - 2b^* = 0$

$3 = 2b^*$

$b^* = \frac{3}{2}$ +2

$f'(b_t) = 4 - 4b_t$ +2

$|f'(0)| = |4 - 4(0)| = |4| > 1 \therefore$ unstable

$|f'(\frac{3}{2})| = |4 - 4(\frac{3}{2})| = |-2| = 2 > 1 \therefore$ unstable +2

6. Given $f(2) = -1, f'(2) = 0, and\ f''(2) = -1$, classify $f(2) = -1$ as a local maximum, a local minimum or say if you are unable to make this determination based on the information provided.

local maximum

7. Find the interval(s) for which $f(x) = x^3 - 3x^2 - 45x$ is concave up.

$f'(x) = 3x^2 - 6x - 45$

$f''(x) = 6x - 6 = 0$

$x = 1$ +6

$f(x)$ is concave up on $(1, \infty)$ +4

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8. Given $f(x) = 1 - e^{-x}$. Compute $\frac{d}{dx}(f^{-1}(0))$ using the definition $\frac{d}{dx}(f^{-1}(x)) = \frac{1}{f'(f^{-1}(x))}$. [Note: $f(0) = 0$]

$f'(x) = -e^{-x}(-1) = e^{-x}$ +2

+3 $f^{-1}(0) = 0$

$$(f^{-1})'(0) = \frac{1}{f'(f^{-1}(0))} = \frac{1}{f'(0)} = \frac{1}{e^{-0}} = \frac{1}{1} = 1$$

or

Find $f^{-1}(x)$: $y = 1 - e^{-x}$

$y - 1 = -e^{-x}$

$1 - y = e^{-x}$

$\ln(1-y) = -x$

$-\ln(1-y) = x$

$f^{-1}(x) = -\ln(1-x)$ +4

$(f^{-1})'(x) = \frac{-1}{1-x} \cdot -1 = \frac{1}{1-x}$ +2

$(f^{-1})'(0) = \frac{1}{1-0} = 1$ +2

or $(f^{-1})'(x) = \frac{1}{e^{-(-\ln(1-x))}} = \frac{1}{e^{\ln(1-x)}} = \frac{1}{1-x}$

$\frac{\frac{3}{2}}{\frac{1}{2}} = \frac{3}{2} \cdot \frac{2}{1} =$

9. Verify that $f(x) = \frac{x+1}{x}$ satisfies the conditions of the Mean Value Theorem on the interval [1/2,2] and find a value of c that is guaranteed to exist.

+2 $f(x)$ is cont on $[\frac{1}{2}, 2]$ and differentiable

$$f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{f(2) - f(\frac{1}{2})}{2 - \frac{1}{2}} = \frac{\frac{3}{2} - 3}{1.5} = \frac{-1.5}{1.5} = -1$$ +2

$$f'(x) = \frac{x(1) - (x+1)(1)}{x^2} = \frac{x - x - 1}{x^2} = \frac{-1}{x^2}$$ +2

$\frac{-1}{x^2} = -1$ +2 $\rightarrow \frac{1}{x^2} = 1 \rightarrow x^2 = 1 \quad x = \pm 1$

only $\boxed{x = 1}$ is in the interval $[\frac{1}{2}, 2]$ +2

10. Use L'Hôpital's Rule to determine the following limit and decide if the function in the numerator or the function in the denominator reaches its limit faster:

$$\lim_{x \to \infty} \frac{\ln(x) + 10}{\sqrt{x}} \overset{LH}{=} \frac{\infty}{\infty} \quad \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{2}x^{-\frac{1}{2}}} = \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \lim_{x\to\infty} \frac{1}{x} \cdot \frac{2\sqrt{x}}{1} = \lim_{x\to\infty} \frac{2}{\sqrt{x}} = 0$$ +4

$\therefore \sqrt{x} \to \infty$ faster than $\ln(x) + 10$ as $x \to \infty$ +4