Math 1290 – Fall 2007 – Exam 3 – May 1st, 2008 Name: Key

1. Use the tangent line approximation to estimate $(4.01)^2$. Is this estimation an over or under approximation? Justify your answer with a sketch. $f(x) = x^2$ $a = 4$ $f'(x) = 2x$

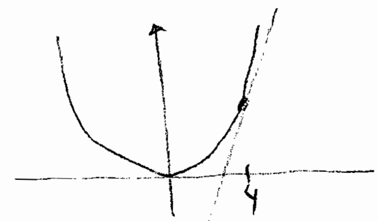
+2 $\hat{f}(x) = f(a) + f'(a)(x-a)$

$\hat{f}(x) = f(4) + f'(4)(x-4) = 16 + 8(x-4)$ +3

$\hat{f}(4.01) = 16 + 8(4.01-4)$

$= 16 + 8(0.01)$

+2 $= 16.08$



+3 under approximation because the tangent line lies below the curve

2. Evaluate the following: (5 pts each)

a) $\int x\sqrt{x^2-4}\,dx = \int \sqrt{u}\,\frac{1}{2}du = \frac{1}{2}\int u^{\frac{1}{2}}du$ +1

$= \frac{1}{2}\cdot\frac{2}{3}u^{\frac{3}{2}} + C$ +1

$= \frac{1}{3}(x^2-4)^{\frac{3}{2}} + C$ +1

$u = x^2 - 4$

$\frac{du}{dx} = 2x$

$\frac{1}{2}du = x\,dx$ +2

b) $\int \sin(3x) - 3x + 3 - \frac{3}{x}\,dx = -\frac{1}{3}\cos(3x) - \frac{3}{2}x^2 + 3x - 3\ln|x| + C$

(+2, +1, +1, +1)

note

$\int \sin(3x)dx = \int \sin(u)\frac{1}{3}du$

$u = 3x$

$\frac{du}{dx} = 3$

$\frac{1}{3}du = dx$

$= \frac{1}{3}\int \sin(u)du$

$= -\frac{1}{3}\cos(3x)$

c) $\int xe^{-2x}dx = x\left(-\frac{1}{2}e^{-2x}\right) - \int -\frac{1}{2}e^{-2x}dx$ +1 $= -\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}dx$ +1

$= -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C$ +1

$u = x$ $dv = e^{-2x}dx$

$du = dx$ $v = -\frac{1}{2}e^{-2x}$ +2

d) $\int_2^\infty \frac{2}{x^3}dx = \lim_{t\to\infty}\int_2^t 2x^{-3}dx$ +2 $= \lim_{t\to\infty} \left.\frac{2x^{-2}}{-2}\right]_2^t$ +1

$= \lim_{t\to\infty} \left.\frac{-1}{x^2}\right]_2^t$

$= \lim_{t\to\infty} \frac{-1}{t^2} - \frac{-1}{2^2} = \frac{1}{4}$ +1

($\frac{-1}{t^2} \to 0$; +1)

$\therefore$ convergent.

3. Standing on the ground, I toss a ball upward with a velocity of 15 meters per second. When does it hit the ground? Given that the acceleration due to gravity $a(t) = -9.8\ \frac{m}{s^2}$. Hint: $a(t) = \frac{dv}{dt}$ *where* $v(t) = velocity$.

$p(0) = 0 \qquad v(0) = 15$

$v(t) = \int a(t)\,dt = \int -9.8\,dt = -9.8t + C$

$v(0) = -9.8(0) + C = 15 \Rightarrow C = 15$

+3 $v(t) = -9.8t + 15$

$p(t) = \int v(t)\,dt = \int -9.8t + 15\,dt = -9.8\frac{t^2}{2} + 15t + C$

$p(0) = -\frac{9.8}{2}(0)^2 + 15(0) + C = 0 \Rightarrow C = 0$

3 $p(t) = -\frac{9.8}{2}t^2 + 15t = -4.9t^2 + 15t$

+2 hits the ground when $p(t) = 0$ $\quad -4.9t^2 + 15t = 0$

$t(-4.9t + 15) = 0$

$t = 0 \quad -4.9t + 15 = 0 \Rightarrow t = \frac{-15}{-4.9} = 3.0611$ +2

4. The rate of growth of a sunflower is defined by $\frac{dS}{dt} = 2 - t + 0.5t^2$ cm per week. If time is measured in weeks and length is measure in centimeters, how much does the sunflower grow during the second month (weeks 5-8)? Please label your solution with the correct units.

+3 $\int_5^8 2 - t + 0.5t^2\,dt = 2t - \frac{1}{2}t^2 + \frac{0.5}{3}t^3 \Big]_5^8$ +3

$= 2(8) - \frac{1}{2}(8)^2 + \frac{1}{6}(8)^3 - \left(2(5) - \frac{1}{2}(5)^2 + \frac{1}{6}(5)^3\right)$ +2

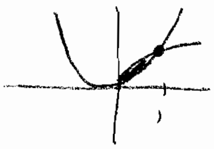
$= 51$ cm +2

5. Verify $y(x) = 1 - e^{-2x}$ is the solution to the differential equation $\frac{dy}{dx} = 2(1 - y)$.

+5 LHS: $\frac{dy}{dx} = -e^{-2x}(-2) = 2e^{-2x}$

+5 RHS: $2(1-y) = 2(1-(1-e^{-2x})) = 2(1-1+e^{-2x}) = 2e^{-2x}$

6. Find the area of the region bounded by the curves $y = x^2$ and $y = \sqrt{x}$.



+5 $A = \int_0^1 \sqrt{x} - x^2 \, dx = \int_0^1 x^{\frac{1}{2}} - x^2 \, dx$

+3 $= \frac{2}{3} x^{\frac{3}{2}} - \frac{1}{3} x^3 \Big]_0^1$

$= \frac{2}{3}(1)^{\frac{3}{2}} - \frac{1}{3}(1)^3 - (0) = \frac{2}{3} - \frac{1}{3} = \boxed{\frac{1}{3}}$

+2

7. Use Euler's method with $\Delta t = 1$ to approximate $P(4)$, given $\frac{dP}{dt} = 2P^2 + 1$ and $P(0)=2$.

$\hat{P}(1) = P(0) + P'(0)\Delta t = 2 + (2(2)^2 + 1)1 = 11$

$\hat{P}(2) = \hat{P}(1) + P'(1)\Delta t = 11 + (2(11)^2 + 1)(1) = 254$

$\hat{P}(3) = \hat{P}(2) + P'(2)\Delta t = 254 + (2(254)^2 + 1)(1) = 129{,}287$

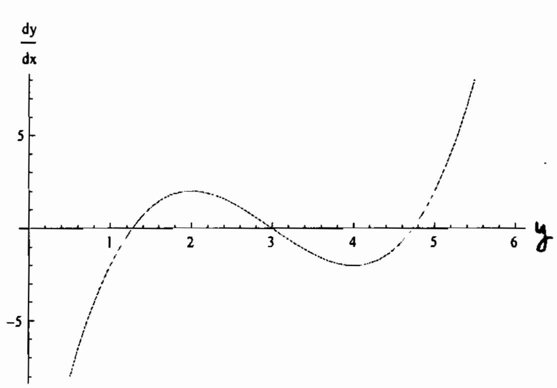
−6 if treated like pure time.

8. Suppose the differential equation $\frac{dG}{dt} = \frac{2G}{e^{-4t}}$ describes the rate of growth a population of geese. If the population is 10 on day 3, is the population growing or shrinking at this time?

$\frac{dG}{dt} = \frac{2(10)}{e^{-4(3)}} = \frac{20}{e^{-12}} = 3255095.828 > 0$

∴ population is growing!

9. From the graph of the differential equation provided sketch the phase line diagram and a plot of the solution if the initial condition is P(0)=2. Please label your axes correctly.



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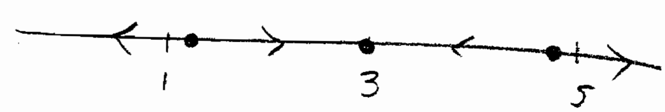

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