Math 1297 – Quiz 5 Name: _____ Sec: ___

Calculate the following integrals:

1. (2 pts)

$\int \frac{x+2}{x^3} dx$

$= \int \left(\frac{1}{x^2} + \frac{2}{x^3} \right) dx$

$= -\frac{1}{x^2} - \frac{1}{x^2} + C$

2. (2 pts)

$\int \sqrt{x} \cos(1+x^{\frac{3}{2}}) dx = \int \cos u \cdot \frac{2}{3} du = \frac{2}{3} \sin(1+x^{3/2}) + C$

$u = 1 + x^{3/2}$

$\frac{du}{dx} = \frac{3}{2} x^{\frac{1}{2}} \qquad \frac{2}{3} du = \sqrt{x}\, dx$

3. (3 pts)

$\int \frac{x+x^3}{\sqrt{9-x^4}} dx$

$= \int \frac{x\, dx}{\sqrt{9-x^4}} + \int \frac{x^3\, dx}{\sqrt{9-x^4}} = \boxed{\frac{1}{2} \sin^{-1}\left(\frac{x^2}{3}\right) - \frac{1}{2}\sqrt{9-x^4} + C}$

$u_2 = 9 - x^4 \qquad \int \frac{-\frac{1}{4}}{\sqrt{u_2}} du_2 = \left(-\frac{1}{4}\right) \cdot 2\sqrt{u_2} = \left(-\frac{1}{2}\right)\sqrt{9-x^4}$

$u_1 = x^2 \qquad \frac{du_1}{dx} = 2x$

$\int \frac{x\, dx}{\sqrt{9-x^4}} = \int \frac{1}{\sqrt{9-u_1^2}} \frac{1}{2} du_1$

$= \frac{1}{2} \sin^{-1}\left(\frac{u_1}{3}\right) = \frac{1}{2} \sin^{-1}\left(\frac{x^2}{3}\right)$

4. (3 pts)

$\int_0^{\ln(2)} e^x \sqrt{1+e^x} dx = \int_2^3 \sqrt{u}\, du$

$= \frac{2}{3} u^{3/2} \Big|_{u=2}^{3}$

$= \frac{2}{3} \left(3^{3/2} - 2^{3/2}\right)$

$u = 1 + e^x$

$\frac{du}{dx} = e^x$

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Calculate the following integrals:

1. (2 pts)

$$\int \frac{x^3+2x+2}{x}\,dx$$

$$= \int \left(x^2 + 2 + \frac{2}{x}\right) dx$$

$$= \frac{1}{3}x^3 + 2x + 2\ln|x| + C$$

2. (2 pts)

$$\int \frac{\cos(1+\sqrt{x})}{\sqrt{x}}\,dx = \int 2\cos u\,du = 2\int \cos u\,du$$

$$= 2\sin(1+\sqrt{x}) + C$$

$u = 1+\sqrt{x}$

$$\frac{du}{dx} = \frac{1}{2}\frac{1}{\sqrt{x}} \Rightarrow 2\,du = \frac{dx}{\sqrt{x}}$$

3. (3 pts)

$$\int \frac{1}{x^2-8x+25}\,dx$$

$$= \int \frac{1}{x^2-8x+4^2+25-4^2}\,dx$$

$$= \int \frac{1}{(x-4)^2+9}\,dx = \int \frac{1}{u^2+3^2}\,du = \frac{1}{3}\tan^{-1}\left(\frac{x-4}{3}\right) + C$$

$u = (x-4)$

4. (3 pts)

$$\int_0^{\ln(2)} \frac{e^x}{1+e^x}\,dx = \int_2^3 \frac{1}{u}\,du = \ln u\Big|_{u=2}^{3} = \ln(3/2)$$

$u = 1+e^x$

$$\frac{du}{dx} = e^x$$

$$du = e^x\,dx$$

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Calculate the following integrals:

1. (2 pts)

$\int \frac{x+1}{x^2} dx$

$= \int \left(\frac{1}{x} + \frac{1}{x^2} \right) dx$

$= \ln x - \frac{1}{x} + C$

2. (2 pts)

$\int \sqrt{x} \sin(1+x^{\frac{3}{2}}) dx = \int (\sin u) \cdot \frac{2}{3} du = \frac{2}{3} \int \sin u \, du$

$= -\frac{2}{3} \cos(1+x^{3/2}) + C$

$u = 1 + x^{3/2}$

$\frac{du}{dx} = \frac{3}{2} x^{\frac{1}{2}}$

$\frac{2}{3} du = \sqrt{x} \, dx$

3. (3 pts)

$\int \frac{x+3x^3}{9+x^4} dx$

$= \int \frac{x}{9+x^4} dx + \int \frac{3x^3}{9+x^4} dx$

$u_1 = x^2$

$\frac{du_1}{dx} = 2x \quad \frac{1}{2} du_1 = x \, dx$

$\int \frac{x}{9+x^4} dx = \int \frac{1}{3^2+u_1^2} \cdot \frac{1}{2} du_1 = \frac{1}{2} \int \frac{1}{3^2+u_1^2} du_1$

$= \frac{1}{2} \cdot \frac{1}{3} \tan^{-1}(x^2) + C$

$= \frac{1}{6} \tan^{-1}(x^2) + C$

$u_2 = 9 + x^4 \quad \frac{du_2}{dx} = 4x^3$

$\int \frac{3x^3}{9+x^4} dx = \int \frac{3}{u_2} \cdot \frac{1}{4} du_2 = \frac{3}{4} \ln u_2 + C$

$= \frac{1}{6} \tan^{-1}(x^2) + \frac{3}{4} \ln(9+x^4) + C$

4. (3 pts)

$\int_0^{\ln(2)} e^x \sqrt{1+e^x} dx = \int_2^3 \sqrt{u} \, du = \frac{2}{3} u^{3/2} \Big]_{u=2}^{3}$

$= \frac{2}{3} (3^{3/2} - 2^{3/2})$

$u = 1 + e^x$

$\frac{du}{dx} = e^x$

$du = e^x \, dx$