2. \( f(x) \) answers will vary:
- a) critical point
- b) \( f'(x) > 0 \)
- c) \( f'(x) < 0 \)
- d) \( f''(x) > 0 \)
- e) \( f''(x) < 0 \)
- f) \( f''(x) = 0 \)

4. \( g(x) \)

6. ans may vary need a function w/ a positive, increasing derivative

8. ans may vary need a function w/ a negative, decreasing derivative

12. \( f(z) = 3z^3 + 2z^2 \)
   - \( f'(z) = 9z^2 + 4z \)
   - \( f''(z) = 18z + 4 \)

14. \( p(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} \)
   - \( p'(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 \)
   - \( p''(x) = 1 + x + \frac{1}{2}x^2 \)

16. \( R(s) = (1 + s^2)(2 + s) = 2 + 2s^2 + s + s^3 \)
   - \( R'(s) = 3s^2 + 4s + 1 \)
   - \( R''(s) = 6s + 4 \)

27. \( y'' = x^9 \)
   - \( y''(x) = 0 \)
5) \[ p(t) = -37t^2 + 30t + 500 = 500 \text{ m} \]

\[ f(t) = -6t + 60 \]

\[ f'(t) = -6 + 60 = 54 \text{ m/s} \]

\[ f''(t) = -6 \text{ m/s}^2 \]

The acceleration is much longer than on Earth.

42. most quickly \( \approx 10 \) second, accelerating most quickly \( \approx 8 \) second

decaying most quickly \( \approx 6 \) second

44. \[ f(b_t) = 2b_t (1 - \frac{b_t}{1000})b_t = 2b_t^2 (1 - \frac{b_t}{1000}) \]

\[ f'(b_t) = 2b_t^2 \left( -\frac{1}{1000} \right) + (1 - \frac{b_t}{1000})(4b_t) = \frac{-b_t^2}{500} + 4b_t - \frac{b_t}{300} = \frac{-3b_t^2}{500} + 4b_t \]

\[ f''(b_t) = \frac{-6}{500} b_t + 4 \]

find critical points:

\[ f'(b_t) = 0 \quad \frac{-3b_t^2}{500} + 4b_t = 0 \quad b_t \left( \frac{-3b_t}{500} + 4 \right) = 0 \]

\[ b_t = 0 \quad \frac{-3b_t^2}{500} + 4 = 0 \quad y = \frac{3b_t}{500} = \frac{2000}{3} \quad b_t \approx 66 \]

\[ f''(b_t) = \frac{-6}{500} b_t + 4 = 0 \quad b_t = 333 \frac{3}{5} \]

\[ TP \]

\[ CP \]

\[ b_t \quad f(b_t) \]

0 \quad 0

333 \quad 148,146

666 \quad 26,296

1000 \quad 20
4. \( h(x) = (2-x^2) e^x \)

\[ h'(x) = (2-x^2) e^x + e^x (-2x) = e^x (2-x^2-2x) \]

\[ h''(x) = e^x (-2x-2) + (2-x^2-2x)e^x = e^x (-2x-2+2-x^2-2x) = e^x (-x^2-4x) \]

6. \( G(z) = \frac{e^z}{z^2} \)

\[ G'(z) = \frac{z^2 e^z - e^z (2z)}{z^4} = \frac{e^z (z^2-2z)}{z^4} \]

\[ G''(z) = \frac{z^4 (e^z (2z-2)-(z^2-2z)e^z) - e^z (z^2-2z)(4z^3)}{z^8} = \frac{e^z (z^4 (2z-2) - 4z^3 (z^2-2z))}{z^8} \]

\[ = \frac{e^z (z^6 - 2z^4 - 4z^5 + 8z^4)}{z^8} = \frac{e^z (z^4 - 4z^5 + 6z^4)}{z^8} \]

\[ = e^z \left( z^4 - 4z^5 + 6z^4 \right) \]

17. \( f(x) = x^2 \ln x \)

\[ f'(x) = x^2 \left( \frac{1}{x} \right) + \ln(x) \cdot 2x = x + 2x \ln x \]

\[ f''(x) = 1 + 2x \cdot \frac{1}{x} + \ln(x) \cdot 2 = 1 + 2 + 2 \ln x = 3 + 2 \ln x \]

18. \( n(x) = \ln(x^2 e^x) = \ln(x^2) + \ln(e^x) = 2 \ln x + x \)

\[ n'(x) = 2 \left( \frac{1}{x} \right) + 1 = \frac{2}{x} + 1 = \frac{2x - 1}{x} \]

\[ n''(x) = -2x^{-2} \]
20. \( f(x) = (2-x)e^x \), \(-2 \leq x \leq 1\)

\[ g'(x) = (2-x)e^x + e^x(-1) = e^x(2-x-1) = e^x(-1-x) \]

\[ g''(x) = e^x(-1) + (1-x)e^x = e^x(-1+1-x) = -xe^x \]

\[ g'(x) = 0 \quad g''(x) = 0 \quad x = \frac{5}{3} \]

\[ e^{(1-x)} = 0 \quad -xe^x = 0 \quad -x = 0 \quad x = 0 \]

\[ x = 0 \]

\[ x = 1 \]

27. \( h(x) = (ax+b)e^x \), \( a = 1 \), \( b = 1 \)

\[ h(x) = (x+1)e^x \]

\[ h'(x) = (x+1)e^x + e^x(1) = e^x(x+1+1) = e^x(x+2) \]

\[ h''(x) = e^x(1) + (x+2)e^x = e^x(1+1+2) = e^x(x+3) \]

28. \( h^{(n)}(x) = e^x(x+11) \)

36. \( P(t) = 10e^t \)

\[ M(t) = P(t) \cdot m(t) \]

\[ M(t) = 10e^t(1-t) \]

\[ m'(t) = 10e^t(-1) + (1-t)(10e^t) = 10e^t(-1+t-1) = -10te^t \]

\[ m''(t) = -10te^t + e^t(-10) = -10te^t - 10e^t = -10e^t(t+1) \]

\[ m'(t) = 0 \quad t = 0 \]

\[ m''(t) = 0 \quad t = 1 \]

No, the total mass, \( M(t) \)

is decreasing for \( t > 0 \).

The mass is zero for \( t = 1 \).
42. \( \frac{db}{dt} = b(t) \) \( b(t) = e^t \)

The differential equation says that the rate of change of the population is equivalent to the size of the population. The solution \( b(t) = e^t \) says that the population grows exponentially, as a function of time.

LHS: \( \frac{db}{dt} = b'(t) = e^t \)

RHS: \( b(t) = e^t \)

The solution is an increasing function (Its first derivative is positive for all \( t \))