2. \( e^x - 3x^2 = 0 \) \( 0 \leq x \leq 1 \)

\( f(x) = e^x - 3x^2 \) is the sum of 2 continuous functions and thus continuous.

\[ f(0) = e^0 - 3(0)^2 = 1 \]

\( f(1) = e^1 - 3(1)^2 = e^1 - 3 = -0.28 \]

Value Theorem: There exists a solution in \([0, 1]\).

4. \( e^x + x^2 - 2 = \cos(2\pi x) - 1 \)

\( f(x) = e^x + x^2 - \cos(2\pi x) - 1 \) is the sum of multiple continuous functions and thus continuous.

\[ f(0) = e^0 + 0^2 - \cos(2\pi(0)) - 1 = -1 \]

\[ f(1) = e^1 + 1^2 - \cos(2\pi(1)) - 1 \approx 1.71 \]

Change in sign, thus by the IVT there exists a solution in \([0, 1]\).

12. \( f(x) = x^2 \) \( x = 0 \) to \( x = 2 \)

* \( f(x) \) is continuous on \([0, 2]\)

\[ f'(x) = 2x \]

\[ f'(c) = \frac{f(b) - f(a)}{b-a} = \frac{f(2) - f(0)}{2-0} = \frac{4-0}{2} = 2 \]

\[ f'(c) = 2c = 2 \]

\[ c = 1 \]

14. \( f(x) = \sqrt{x} \) \( x = 0 \) to \( x = 2 \)

* \( f(x) \) is continuous on \([0, 2]\)

\[ f'(x) = \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}} \]

\[ f'(c) = \frac{f(b) - f(a)}{b-a} = \frac{\sqrt{2} - 0}{2-0} = \frac{\sqrt{2}}{2} \]

\[ f'(c) = \frac{1}{2\sqrt{c}} = \frac{\sqrt{2}}{2} \rightarrow 2\sqrt{c} = 2 \]

\[ \sqrt{c} = \frac{2}{2\sqrt{2}} \]

\[ c = \frac{1}{2} \]
22. \( g(x) = 1 \times 1 \) \( g(x) \) is continuous for all real numbers. Yes, it satisfies the conditions of the Intermediate Value Theorem.

\[
\frac{g(-1) - g(2)}{-1 - 2} = \frac{1 - 4}{-3} = \frac{3}{3}
\]

But the slopes of the tangent lines are \(-1\) for \( x < 0 \) and 1 for \( x > 0 \). There is never a value of \( x \) for which \( f'(x) = \frac{1}{3} \). However, this does not contradict the Mean Value Theorem because its conditions are not satisfied. Specifically, \( f(x) \) is not differentiable at \( x = 0 \).

30. \( c_{41} = 0.25 e^{-3c_t} c_t + 0.75\delta \) where \( \delta = 5 \)

\( f(c_t) = 0.25 e^{-3c_t} c_t + 3.75 \) * note \( f(c_t) \) is a continuous function

\( f(0) = 0.25 e^{3(0)}(0) + 3.75 = 3.75 \) where \( c_{41} > c_t \)

\( f(5) = 0.25 e^{-3(5)}(5) + 3.75 = 3.75 \) where \( c_{41} < c_t \)

Since equilibrium points occur where \( c_t = c_{41} \), then between \( c_t = 0 \) and \( c_t = \delta = 5 \), the updating function must cross the diagonal.

\[ c_t = 0.25 e^{-3c_t} c_t + 3.75 \rightarrow f(c_t) = 0.25 e^{-3c_t} c_t + 3.75 - c_t \]

\( f(0) = 0.25 e^{3(0)}(0) + 3.75 - 0 = 3.75 \) and \( f(5) = 0.25 e^{-3(5)}(5) + 3.75 - 5 = -1.25 \)

Since there is a change of sign, by TVT there must be a solution between \( c_t = 0 \) and \( c_t = 5 \).

According to the Mean Value Theorem the speed (instantaneous velocity) at some point during the hour must be equal to the average velocity (40 mph). According to the Intermediate Value Theorem the instantaneous speed velocity must take on every velocity from 20 to 60 mph during the hour.
2. \( \lim_{x \to \infty} \ln \left( \frac{x}{5} \right) = \infty \)

4. \( \lim_{x \to \infty} 1 - e^{-4x} = 1 \) because \( \lim_{x \to \infty} e^{-4x} = 0 \)

8. \( \lim_{x \to \infty} (x^2 + 2)^{0.23} = \infty \)

10. \( \begin{array}{c|c|c|c}
   x & x^3 & 1000x & x^3 \to \infty \text{ faster than } 1000x \text{ as } x \to \infty \\
   1 & 1 & 1000 & \\
   10 & 1000 & 10000 & \text{The order switches as } x \text{ gets larger} \\
   100 & 1,000,000 & 100,000 & x^3 = 1000x \\
   & x^2 = 1000 & x = 31.62 \text{ (So, for } x > 31.61) \end{array} \)

14. \( \begin{array}{c|c|c|c|c|c|c}
   x & 10x^{0.1} & x^{0.5} & x^{0.5} \to \infty \text{ faster than } 10x^{0.1} \text{ as } x \to \infty \\
   1 & 1 & 1 & \\
   10 & 10 & 3.16 & \text{The order switches as } x \text{ gets larger} \\
   100 & 15.8 & 10 & 10x^{0.1} = x^{0.5} \Rightarrow (10)^{x^{0.4}} (x^{0.4})^{x^{0.1}} \\
   & x^{0.1} & x^{0.1} & x = 316.2 \text{ (So, for } x > 316.2) \end{array} \)

18. \( \begin{array}{c|c|c|c|c|c|c|c|c}
   x & x^{-0.1} & 25x^{-0.2} & 25x^{-0.2} \to 0 \text{ faster than } x^{-0.1} \text{ as } x \to \infty \\
   1 & 1 & 25 & \text{Rule: } ax^{-n} \to 0 \text{ faster for larger } n \\
   10 & 0.79 & 15.7 & \text{The order switches as } x \text{ gets larger} \\
   100 & 0.03 & 9.95 & \frac{x^{-0.1}}{x^{-0.2}} = 25x^{-0.2} \\
   & x^{-0.2} & x^{-0.2} & \\
   & x^{0.1} = 25 & \text{So, for } x > 9.5 \times 10^{13} \\
   & x = (25)^{0.1} \\
   & x = 9.5 \times 10^{13} \end{array} \)
22. \( \lim_{{c \to \infty}} \frac{Ac}{{\ln(1+c)}} = \infty \)

24. \( \lim_{{c \to \infty}} \frac{e^2}{1 + 10c} = \infty \)

30. \( \alpha(c) = \frac{5c}{1 + c} \)

\( \alpha'(c) = \frac{(1+c)(5) - (5c)(1)}{(1+c)^2} = \frac{5+5c-5c}{(1+c)^2} = \frac{5}{(1+c)^2} \)

\( \alpha'(0) = \frac{5}{(1+0)^2} = 5 \)

\( \lim_{{c \to \infty}} \alpha'(c) = \lim_{{c \to \infty}} \frac{5}{(1+c)^2} = 0 \)

Yes, the curve starts with a positive rate of change that decreases (levels off) to zero.

32. \( b_{t+1} = r b_t \quad b_t = b_0 r^t \)

\( 10^8 (1.5)^t = 10^{10} \)

\( t \ln(1.5) = \ln(10^2) \)

\( t = \frac{\ln(10^2)}{\ln(1.5)} = 11.36 \)

34. \( b_{t+1} = 0.96 b_t \quad b_t = 10^8 (0.9)^t \)

\( \lim_{{t \to \infty}} b_t = \lim_{{t \to \infty}} 10^8 (0.9)^t = 0 \)

\( (0.9)^t = \frac{10^8}{10^5} = 10^3 \)

\( t \ln(0.9) = \ln(10^3) \)

\( t = \frac{\ln(10^3)}{\ln(0.9)} = 109.27 \)

37. \( m_{t+1} = 0.5 m_t + 1 \)

38. \( \lim_{{t \to \infty}} m_t = \lim_{{t \to \infty}} 2 + (0.5)^t = 2 \)

0.5 \( m^* = 1 \quad m^* = 2 \)

39. \( 1% \quad \frac{2}{3} = 0.02 \quad 2.02 = 2 + (0.5)^t \quad \ln\left(\frac{0.02}{3}\right) = t \ln(0.5) \)

\( 0.02 = (0.5)^t(3) \quad t = \frac{\ln\left(\frac{0.02}{3}\right)}{\ln(0.5)} \)

\( \frac{0.02}{3} = (0.5)^t \quad t \approx 7.2 \)