20. \( \frac{db}{dt} = 3b \quad b(0) = 10 \quad \Delta t = 0.5 \)
\[ b(0.5) = b(0) + b'(0) \Delta t = 10 + 3(10)(0.5) = 25 \]
\[ b(1) = b(0.5) + b'(0.5) \Delta t = 25 + 3(25)(0.5) = 62.5 \]
Exact answer from #16 \( b(1) = 10e^{3t} \quad b(1) = 10e^{3(1)} = 300.85 \)
Euler's method provides a very low estimate.

20. \[ \frac{dz}{dt} = \frac{1}{z-1} \quad z(0) = 2 \quad \Delta t = 1 \]
\[ z(1) = z(0) + z'(0) \Delta t = 2 + \left( \frac{1}{2-1} \right)(1) = 3 \]
\[ z(2) = z(1) + z'(1) \Delta t = 3 + \left( \frac{1}{3-1} \right)(1) = 3.5 \]
\[ z(3) = z(2) + z'(2) \Delta t = 3.5 + \left( \frac{1}{3.5-1} \right)(1) = 3.9 \]
\[ z(4) = z(3) + z'(3) \Delta t = 3.9 + \left( \frac{1}{3.9-1} \right)(1) = 4.25 \]
Exact answer from #18: \( z(4) = 1 + \sqrt{1+2+3} \) \( z(4) = 1 + \sqrt{6} \approx 4.47 \)
Here, Euler's method is close.

28. \( \frac{db}{dt} = \lambda(b) ; \quad \lambda(0) = 4 \quad \text{slope} \; m = -0.001 \quad \lambda(b) = -0.001b + 4 \)
\[ \frac{db}{dt} = (-0.001b + 4) b = -0.001b^2 + 4b \]
\( b = 1000 \quad \frac{db}{dt} = -0.001(1000)^2 + 4(1000) = 3000 \) The population, \( b(t) \), is increasing.
\( b = 5000 \quad \frac{db}{dt} = -0.001(5000)^2 + 4(5000) = -5000 \) The population, \( b(t) \), is decreasing.

29. \( \lambda(b) = -2 \quad \text{slope} \; m = 0.01 \quad \lambda(b) = 0.01b - 2 \)
\[ \frac{db}{dt} = (0.01b - 2) b = 0.01b^2 - 2b \]
\( b = 100 \quad \frac{db}{dt} = 0.01(100)^2 - 2(100) = -200 \) The population, \( b(t) \), is decreasing.
\( b = 300 \quad \frac{db}{dt} = 0.01(300)^2 - 2(300) = 300 \) The population, \( b(t) \), is increasing.

30. \[ \frac{dC}{dt} = \beta \Gamma \quad \text{The concentration increases.} \]
\[ \frac{dC}{dt} = 38C - BC \]

If \( C = 1 \), then \( \frac{dC}{dt} = 38C - BC = 2 \beta C > 0 \)

So, the concentration in the cell would be increasing and would become larger than the concentration outside the cell.

If \( C = 1 \) then \( \frac{dC}{dt} = -\frac{1}{2} \beta C - BC = -\frac{1}{2} \beta C < 0 \)

So, the concentration in the cell is decreasing and will become lower than the concentration outside.

\[ \frac{dH}{dt} - \alpha (A - H) \]

\[ H(t) = A + (H(0) - A)e^{-\alpha t} \]

a) \( H(t) = 30 + (40-30)e^{-0.02t} = 30 + 10e^{-0.02t} \)

LHS: \( \frac{dH}{dt} = H'(t) = 10e^{-0.02t} (-0.02) = -0.2e^{-0.02t} \)

RHS: \( \alpha (A - H) = 0.02 (30 - (30 + 10e^{-0.02t})) = 0.02 (-10e^{-0.02t}) = 0.2e^{-0.02t} \)

Initial condition \( H(0) = 40 \)

\[ H(0) = 30 + 10e^{-0.02(0)} = 30 + 10 = 40 \]

b) \( H(1) = 30 + 10e^{-0.02(1)} = 29.8 \) \( \text{°C} \)

\[ H(2) = 30 + 10e^{-0.02(2)} = 29 \) \( \text{°C} \)

\[ \alpha = 0.02 \]

\[ A = 30, H(0) = 40 \]

\[ \Delta t = 1 \]

\[ \frac{dH}{dt} = 0.02 (30 - H(t)) \]

\[ H(0) = H(0) + H'(0) \Delta t = 40 + 0.02 (30 - 40) = 29.8 \) \( \text{°C} \)

\[ H(2) = H(0) + H'(0) \Delta t = 39.8 + 0.02 (30 - 39.8) (1) = 39.6 \) \( \text{°C} \)

\[ C(t) = \Gamma + (C(0) - \Gamma) e^{-\beta t} \]

\[ C(0) = 2 + (5 - 2)e^{-0.1t} = 2 + 3 e^{-0.1t} \]

\[ C(10) = 2 + 3 e^{-0.1(10)} = 3.10 \]

\[ C(20) = 2 + 3 e^{-0.1(20)} = 2.41 \]

\[ C(60) = 2 + 3 e^{-0.1(60)} = 2.01 \]
2. \[ \frac{dx}{dt} = 1 - e^x \]
   \[ \frac{dx}{dt} = 0 \quad \text{when} \quad 1 - e^x = 0 \]
   \[ 1 = e^x \quad \Rightarrow \quad \ln(1) = \ln(e^x) \]
   \[ x = \ln(1) = 0 \]

4)
   \[ \frac{dz}{dt} = \frac{1}{z} - 3 \]
   \[ \frac{dz}{dt} = 0 \quad \text{when} \quad \frac{1}{z} - 3 = 0 \]
   \[ \frac{1}{z} = 3 \]
   \[ z = \frac{1}{3} \]

6)
   \[ \frac{dx}{dt} = cx + x^2 \]
   \[ \frac{dx}{dt} = 0 \quad \text{when} \quad cx + x^2 = 0 \]
   \[ x(c + x) = 0 \]

7. \[ \frac{dw}{dt} = \alpha e^{\beta w} - 1 \]
   \[ \frac{dw}{dt} = 0 \quad \text{when} \quad \alpha e^{\beta w} - 1 = 0 \]
   \[ \alpha e^{\beta w} = 1 \quad \Rightarrow \quad e^{\beta w} = \frac{1}{\alpha} \]
   \[ \beta w = \ln(\frac{1}{\alpha}) \]
   \[ \omega = \frac{\ln(\frac{1}{\alpha})}{\beta} = -\frac{\ln(\alpha)}{\beta} \]

10.

12. Given  \[ 2, 4, 6 \]

14.

\[ \frac{dx}{dt} \]
18. \( \frac{dz}{dt} = \frac{1}{2} - 3 \)

\[ \frac{dz}{dt} = \frac{1}{2} - 3 \]

Phase line diagram

24. \( \frac{db}{dt} = 2b - h \)  \( \lambda = 0.5 \)  \( h = 1000 \)

\[ \frac{db}{dt} = 2b - 1000 = 0 \]

\[ b = \frac{1000}{2} = 500 \]

This population can outgrow the harvest if \( b > 1000 \) and starts at a large enough value (size). If the size starts too small, it will be driven to extinction by the harvest \( b < 2000 \) at \( b = 1000 \) the arrow points right, consistent with an increasing population

at \( b = 5000 \) the arrow points to the left, consistent with a decreasing population

\[ \frac{dc}{dt} = 3b - \beta \Gamma - \beta c = 0 \]

\[ 3b = \beta \Gamma \]

\[ \beta \Gamma = b \]

\[ c = 3b \]

\[ \frac{dc}{dt} = 0.5b - \beta c = 0 \]

\[ 0.5b = \beta c \]

\[ c = 0.5b \]

When \( c = \Gamma \), \( \frac{dc}{dt} < 0 \) This is consistent with a decreasing internal concentration.