Math 1290 HW4  Sec 1.5: 2, 8, 10, 18, 22, 28, 30, 32, 34, 58

9. \[ \psi_{t+1} = \frac{\psi_t}{2} \quad \psi_t = 4 \quad \psi_{t+1} = \frac{4}{2} = 2 \]
   \[ \psi_t = 8 \quad \psi_{t+1} = \frac{8}{2} = 4 \]
   \[ \psi_t = 12 \quad \psi_{t+1} = \frac{12}{2} = 6 \]

8. \[ M_{t+1} = 0.75M_t + 2 \]
   \[ f(M_t) = 0.75M_t + 2 \]
   \[ M_{t+2} = f(f(M_t)) = f(0.75M_t + 2) = 0.75(0.75M_t + 2) + 2 \]
   \[ = 0.5625M_t + 1.5 + 2 = 0.5625M_t + 3.5 \]

Check
   \[ f(f(16)) = 0.5625(16) + 3.5 = 12.5 \]
   \[ f(16) = 0.75(16) + 2 = 14 \rightarrow f(14) = 0.75(14) + 2 = 12.5 \]

10. \[ l_{t+1} = l_t - 1.7 \]
   \[ l_t = l_{t+1} + 1.7 \rightarrow l_0 = l_1 + 1.7 \]
   \[ l_0 = 13.1 + 1.7 = 14.8 \]

18. \[ M_{t+1} = 0.75M_t + 2 \quad M_0 = 16 \]

\[ m = 0.75(16) + 2 = 14 \]
\[ m = 0.75(14) + 2 = 12.5 \]
\[ m = 0.75(12.5) + 2 = 11.375 \]
\[ m = 0.75(11.375) + 2 = 10.53 \]
\[ m = 0.75(10.53) + 2 = 9.9 \]
*note: using the calculator to iterate I can see the solutions are heading toward 8.

\[
\begin{align*}
    m_0 - 8 &= 16 - 8 = 8 = 8(0.75)^0 \\
    m_1 - 8 &= 14 - 8 = 6 = 8(0.75)^1 \\
    m_2 - 8 &= 12.5 - 8 = 4.5 = 8(0.75)^2 \\
    m_3 - 8 &= 11.37 - 8 = 3.375 = 8(0.75)^3
\end{align*}
\]

Thus,

\[
M_t = 8 + (0.75)^t \cdot 8
\]

28. \[M_{20} = 8 + (0.75)^{20} \cdot 8 = 8.0254\]

28. \[P_{t+1} = \frac{P_t}{4} = \frac{1}{2} P_t\]

<table>
<thead>
<tr>
<th>t</th>
<th>(P_t)</th>
<th>(\frac{P_t}{2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
<td>50</td>
</tr>
<tr>
<td>1</td>
<td>60</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>15</td>
</tr>
</tbody>
</table>

If you divide by 4 first and then divide, you get the same result.

Thus, the actions commute.

30. a) lose half then gain 10  
    b) gain 10 then loose half.

<table>
<thead>
<tr>
<th>t</th>
<th>#</th>
<th>t</th>
<th>#</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>1</td>
<td>60</td>
<td>1</td>
<td>55</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>2</td>
<td>32.5</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>3</td>
<td>21.25</td>
</tr>
</tbody>
</table>

These actions do not commute.

32. \[h_t = 10 + t\]

\[h_{100} = 10 + 100 = 110\] meters. This is pretty tall.

58. a) \[b_0 = 3 \times 10^6\]
    b) \[b_{t+1} = 2(b_t - 1 \times 10^6)\]
    c) The population is smaller because the harvesting is done before reproduction. Thus, fewer bacteria are involved in reproduction.
4. \( e_{t+1} = 2t - 1.7 \)

Both the updating function with cobwebbing and the solution are consistent because they decrease without bound.

5. \( m_{t+1} = 0.75m_t + 2 \)

The cobweb diagram shows a solution that decreases towards an equilibrium at \( m_t = 8 \). The graph of the solution seems to be behaving the same.

10. \( x_{t+1} = 4 - x_t \)  \( x_0 = 1 \)

| \( t \) | 0 | 1 | 2 | 3 | 4 | 5 |...
|-------|---|---|---|---|---|---|---
| \( x_t \) | 1 | 3 | 1 | 3 | 1 | 3 |...
\[ x_{t+1} = \frac{x_t}{x_t - 1} \quad x_t \geq 1 \quad x_0 = 3 \]

\[
\begin{array}{c|c|c|c|c|c}
\varepsilon & 0 & 1 & 2 & 3 \\
\hline
x_t & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \\
\end{array}
\]

Stuck in a cycle again.

The equilibrium (fixed point) is at \((5,5)\).

\[
G(y) = y^2 - 1
\]

Find equilibrium: \[ y = y^2 - 1 \]
\[
y^2 - y - 1 = 0
\]

Use quadratic formula with \[ a = 1, b = -1, c = -1 \]
\[
y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)} = \frac{1 \pm \sqrt{5}}{2}
\]

\( \text{But } \frac{1 - \sqrt{5}}{2} \text{ does not make sense.} \)

\[
b_{t+1} = 2b_t - 5 \quad 0 \leq b_t \leq 10
\]

Let \( b^* = b_t = b_{t+1} \)

\[
b^* = 2b^* - 5
\]

\[
-b^* = -5 \Rightarrow b^* = 5
\]

For \( 0 \leq b_t \leq 5 \) the updating function lies below the diagonal and for \( 5 \leq b_t \leq 10 \) the updating function lies above the diagonal.
21. \[ l_{t+1} = l_t + 1.7 + \frac{l_t}{l_t - 1.7} \]

22. \[ l_t \to l^* = l_t = l_{t+1} \]

23. \[ l^* = l^* - 1.7 \]

24. \[ 0 = -1.7 \quad \text{(not possible!)} \quad \text{So, there is no equilibrium and} \]

\[ \text{This corresponds to what was observed in 16.} \]

30. \[ x_{t+1} = \frac{x_t}{x_t - 1} \]

\[ (x^*)^2 - x^* = x^* \]

\[ (x^*)^2 - 2x^* = 0 \]

\[ x^* (x^* - 2) = 0 \]

\[ x^* = 0 \quad \text{or} \quad x^* - 2 = 0 \Rightarrow x^* = 2 \]

\[ \frac{x^*}{x^* - 2} = 0 \]

\[ x^* = 0 \quad \text{or} \quad x^* - 2 = 0 \Rightarrow x^* = 2 \]

\[ \text{Same as the result in 112.}\]

32. \[ x_{t+1} = b - x_t \]

\[ \text{For all values of } b \text{ there will be one equilibrium point and for } b < 0 \]

\[ \text{The equilibrium point will be negative.} \]

\[ x^* = \frac{b}{2} \]

34. \[ y_{t+1} = 2x_t + 30 \quad x_0 = 0 \]

5 steps didn’t quite fit, but it is clear already that no equilibrium will be reached.