1. 1L @ 30°C mixed with 2L @ 100°C
   Total volume = 1 + 2 = 3L
   \[ \text{Temp} = \frac{1}{3}(30) + \frac{2}{3}(100) \approx 76.7°C \]

6. V1 L @ 30°C mixed with V2 L @ 100°C
   Total volume: \( V_T = V_1 + V_2 \)
   \[ \text{Temp} = \frac{V_1}{V_1 + V_2}(30) + \frac{V_2}{V_1 + V_2}(100) \]
   Let \( V_1 = 1 \) L & \( V_2 = 2 \) L
   \[ \text{Temp} = \frac{1}{1+2}(30) + \frac{2}{1+2}(100) \approx 76.7°C \] (same as #1)

12. 100 total

| # | Score | Adjusted Score | Old average: \( \frac{10}{100}(20) + \frac{20}{100}(40) + \frac{30}{100}(60) + \frac{40}{100}(80) \)
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>90</td>
<td>60</td>
<td>( 0.1(20) + 0.2(40) + 0.3(60) + 0.4(80) = 60 )</td>
</tr>
<tr>
<td>20</td>
<td>40</td>
<td>70</td>
<td>New average: ( 0.1(60) + 0.2(70) + 0.3(80) + 0.4(90) = 80 )</td>
</tr>
<tr>
<td>30</td>
<td>40</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>30</td>
<td>90</td>
<td>Yes, the new average is the old average moved halfway to 100</td>
</tr>
</tbody>
</table>

14. \( V = 1L \) \( W = 0.1L \) \( O = 8 \text{mmoles} \) \( C_0 = 4 \text{ mmol/L} \)
   a) \( V \cdot C_0 = 1 \cdot 4 = 4 \text{ mmol/L} \)
   b) \( W \cdot C_0 = 0.1 \cdot 4 = 0.4 \text{ mmol} \)
   c) \((V - W)C_0 = (1 - 0.1)(4) = 0.9(4) = 3.6 \text{ mmol} \)
   d) \( Wδ = 0.1(8) = 0.8 \text{ mmol} \)
   e) \((V - W)C_0 + Wδ = 3.6 + 0.8 = 4.4 \)
   f) \( \frac{4.4}{1} = 4.4 \text{ mmol/L} \)
   g) \( C_{t+1} = (1 - O)C_0 + Oδ = (1 - W/V)C_0 + W/Vδ \)
   \[ C_{t+1} = (1 - 0.1)4 + 0.1(8) = 0.9C_0 + 0.8 \]
   \( C_0 = 4 \) \( C_1 = 0.9(4) + 0.8 = 4.4 \)
18. From Fig. \( C_{t+1} = 0.94 + 0.8 \)

22. \( C_{t+1} = (1-q)C_t + q\delta \) where \( q = \frac{W}{V} = \frac{0.1}{1} = 0.1 \)

\[
C_{t+1} = (1-0.1)C_t + 0.1(8)
\]

\[
C_{t+1} = 0.9C_t + 0.8
\]

Let \( C^* = C_{t+1} = C_t \)

\[
C^* = 0.9C^* + 0.8
\]

\[
0.1C^* = 0.8 \Rightarrow C^* = \frac{0.8}{0.1} = 8
\]

25. \( \delta = 0.2 \) \( q = 0.4 \) \( \alpha = 0.1 \)

**Method 1:** \( C_{t+1} = (1-q)(1-\alpha)C_t + q\delta \)

\[
C_{t+1} = (1-0.4)(1-0.1)C_t + 0.4(0.2) \]

**Initial Condition:** \( C_{t+1} = 0.54C_t + 0.084 \)

**Find eq pt let \( C^* = C_{t+1} = C_t \)**

\[
C^* = 0.54C^* + 0.084
\]

\[
0.46C^* = 0.084 \Rightarrow C^* = \frac{0.084}{0.46} = 0.1826
\]

The equilibrium concentration is higher \( 0.1826 > 0.095 \)

Because more air exchanged with the outside.

Here \( q = 0.4 \) is \( q = 0.2 \) in example.
20. \( q = 0.1 \) \( \Delta = 0.05 \) \( \gamma = 0.21 \)
   (again both methods from #25 are possible)
   
   Method 1:
   
   \[ C_{t+1} = 0.9 (0.95) C_t + 0.021 \]
   The concentration is about the
   
   \[ C_{t+1} = 0.855 C_t + 0.021 \]
   some because \( \frac{1}{2} \) as much is
   
   \[ c^* = 0.855 c^* + 0.021 \]
   being absorbed \( (x = 0.05) \) one of
   
   \[ 0.145 c^* = 0.021 \]
   half as much being breathed
   
   \[ c^* = \frac{0.021}{0.145} = 0.145 \]
   in \( (q = 0.1) \)

   No, gasping gets less oxygen because the fraction absorbed
   is much lower. Also, it takes much longer to reach
   equilibrium in this case.

28. \( \delta = 0.21 \) \( q = 0.2 \) \( C_t \) is reduced by 3\% \( \Rightarrow 0.03 \)

   \[ C_{t+1} = (1-q)(C_t-0.03) + q \delta \]
   
   \[ C_{t+1} = (0.8)(C_t-0.03) + 0.2(0.21) \]
   When \( C_t \) falls below 0.03
   
   \[ C_{t+1} = 0.8 C_t - 0.024 + 0.042 \]
   there is not enough for
   
   \[ C_{t+1} = 0.8 C_t + 0.018 \]
   absorption, So, the function
   
   \[ c^* = C_{t+1} \]
   \[ c^* = 0.8 c^* + 0.018 \]
   \[ 0.2 c^* = 0.018 \]
   \[ c^* = 0.09 \]

36. \( r = 0.2 \) addition of \( 5 \times 10^6 \) after each generation

   a) \( P_0 = 3 \times 10^6 \) \( P_1 = 0.2(3 \times 10^6) = 0.6 \times 10^6 \) bacteria
   
   b) \( 0.6 \times 10^6 + 5 \times 10^6 = 5.6 \times 10^6 \) bacteria
   
   c) \( P_{t+1} = 0.2 P_t + 5 \times 10^6 \)
2. \( B_0 = 800 \quad R_0 = 200 \)

\[
B_{t+1} = \frac{1}{2}(800) \quad R_{t+1} = \frac{1}{2}(200)
\]

\[
B_t = 1600 \quad R_t = 200
\]

Fraction of \( B_t \): \[
\frac{1600}{1600+200} = \frac{1600}{1800} = \frac{8}{9} \quad \text{(Check: } \frac{1}{2} + \frac{1}{2} = \frac{1}{2} = 1)\]

Fraction of \( R_t \): \[
\frac{200}{4600+200} = \frac{200}{4800} = \frac{1}{24}
\]

4. \( B_{t+1} = 5B_t \quad R_{t+1} = R_t = 200 \)

Fraction of \( B_t \): \[
\frac{800}{800+200} = \frac{800}{1000} = \frac{4}{5} \quad \text{(Check: } \frac{800}{800+200} + \frac{200}{800+200} = 1)\]

\[
S = \frac{800}{800+200} = 1.8 \quad r = 1.8 \\
B_t = 2.5 \times 10^5
\]

10. \( B_{t+1} = \frac{5S}{S + (1 - r)} \quad s = 1.8 \quad r = 1.8 \\
B_t = 2.5 \times 10^5
\]

\[
P_t = \frac{1.2 \times 10^5}{1.2 \times 10^5 + 3.5 \times 10^6} = 0.03 \\
m_{t+1} = 0.03(1.2 \times 10^5) = 3.6 \times 10^4 \\
b_{t+1} = 0.03(3.5 \times 10^6) = 1.05 \times 10^7
\]

\[
q_{t+1} = \frac{m_{t+1}}{m_{t+1} + b_{t+1}} = \frac{3.6 \times 10^4}{3.6 \times 10^4 + 1.05 \times 10^7} = 0.03
\]

\[\text{No change in the fraction because both types are reproducing at the same rate (r = 1)}\]

11. \( p_t = \frac{p_t}{p_t + (1 - p_t)} \)

**Method 1: Graphically**

- Plot \( p_t = p_t \)
- Plot \( p^* = \frac{p_t}{p_t + (1 - p_t)} \)
-找到交点

- \( p^* = 0 \) or \( p^* = 1 \)

**Method 2: Algebraically**

- Let \( p^* = p_t = p_{t+1} \)
- \( p^* = \frac{p^*}{p^* + 2(1 - p^*)} \)
- \( p^*(p^* + 2(1 - p^*)) = p^* \)
- \( (p^*)^2 + 2p^* - 2p^* = p^* \)
- \( -p^2 + p^* = 0 \)
- \( p^* - p^* + 1 = 0 \)
- \( p^* = 1 \)
Method 1: Graphically

Method 2: Algebraically, let \( p^* = p_t = p_{t+1} \)

\[
p^* = \frac{4p^*}{4p^* + 0.5(1-p^*)} \]

\[
p^*(4p^* + 0.5 - 0.5p^*) = 4p^* \]

\[
(4p^*)^2 + 0.5p^* - 0.5(4p^*) = 4p^* \]

\[
2.5p^* - 3.5p^* = 0 \]

Fixed points at \( p^* = 0 \) and \( p^* = 1 \)

\[3.5p^*(p^* - 1) = 0 \Rightarrow p^* = 0 \text{ or } p^* = 1\]

Important: Although the set up in #11 & 12 are very different, having the same two fixed points indicates that there are only two possible long-term outcomes, either the mutant population declines to zero or it takes over.

The fixed point at \( x = 1.4 \) is stable (attracting) and the fixed point at \( x = 0.5 \) is unstable (repelling).

33. \[\begin{array}{cccc}
X_t = 100 & Y_t = 100 & X_{t+1} = 110 & Y_{t+1} = 90 \\
-20 & -30 & -22 & -27 \\
80 & 70 & 88 & 63 \\
+30 & +20 & +27 & +22 \\
X_{t+1} = 110 & Y_{t+1} = 90 & X_{t+2} = 115 & Y_{t+2} = 85
\end{array}\]

34. \[\begin{array}{cccc}
X_t = 200 & Y_t = 0 & X_{t+1} = 160 & Y_{t+1} = 40 \\
-40 & +40 & -32 & -12 \\
X_{t+1} = 160 & Y_{t+1} = 40 & 128 & 28 \\
+12 & +32 & 140 & 60
\end{array}\]

35. \[X_{t+1} = X_t - 0.2X_t + 0.3Y_t = 0.8X_t + 0.3Y_t \]

\[Y_{t+1} = Y_t - 0.3Y_t + 0.2X_t = 0.2X_t + 0.7Y_t \]
Because no butterflies dying or reproducing, the total population remains the same for all time.

\[ x_{t+1} + y_{t+1} = x_t + y_t \]

\[ p_{t+1} = \frac{x_{t+1} + y_{t+1}}{x_t + y_t} = 0.8 \frac{x_t + 0.3 y_t}{x_t + y_t} = 0.8 \frac{x_t + 0.3 y_t}{x_t + y_t} + \]

Let \( p_t = \frac{x_t}{x_t + y_t} \). Then \( 1 - p_t = \frac{y_t}{x_t + y_t} \) \[ p_{t+1} = 0.8 p_t + 0.3 (1 - p_t) \]

Find equilibrium. Let \( p^* = p_{t+1} \)

\[ p^* = 0.8 p^* + 0.3 (1 - p^*) \]

\[ 0.5 p^* = 0.3 \]

\[ p^* = 0.6 \]

38. a) \( x_t = 100 \quad y_t = 100 \quad x_{t+1} = 190 \quad y_{t+1} = 160 \)

\[ \begin{array}{c}
-20 \\
80 \\
130 \\
50
\end{array} \begin{array}{c}
30 \\
70 \\
20 \\
+ 150
\end{array} \begin{array}{c}
-3 \frac{3}{3} \\
15 \frac{3}{3} \\
+ 48 \\
+ 33
\end{array} \]

Before migration \( \begin{array}{c}
110 \\
70 \\
190 \\
200
\end{array} \begin{array}{c}
0.5 \\
0.5 \\
0.5 \\
0.5
\end{array} \begin{array}{c}
150 \\
150 \\
150 \\
150
\end{array} \begin{array}{c}
< \text{after migration} \end{array} \]

after migration \( \begin{array}{c}
190 \\
160 \\
110 \\
352
\end{array} \begin{array}{c}
70 \\
70 \\
70 \\
+ 30
\end{array} \begin{array}{c}
152 \\
152 \\
152 \\
260
\end{array} \begin{array}{c}
< \text{after reproduction} \end{array} \]

b) \( x_{t+1} = 2(x_t - 0.2 x_t) + 0.3 y_t \quad y_{t+1} = 2(y_t - 0.3 y_t) + 0.2 x_t \)

\[ x_{t+1} = 1.6 x_t + 0.3 y_t \quad y_{t+1} = 1.4 y_t + 0.2 x_t \]

d) \( \frac{p_{t+1}}{p_t} = \frac{1.6 x_t + 0.3 y_t}{1.8 x_t + 1.7 y_t} = \frac{1.6 x_t + 0.3 y_t}{1.8 x_t + 1.7 y_t} = \frac{1.6 x_t + 0.3 y_t}{1.8 x_t + 1.7 y_t} = \]

\[ p^* = \frac{1.6 p^* + 0.3 (1 - p^*)}{1.8 p^* + 1.7 (1 - p^*)} = \frac{1.6 p^* + 0.3 - 0.3 p^* - 1.3 p^* + 0.3}{1.8 p^* + 1.7 - 1.7 p^* + 0.1 p^* + 1.7} \]

\[ p^*(0.1 p^* + 1.7) = 1.3 p^* + 0.3 \]

\[ 0.1(p^*)^2 + 1.7 p^* = 1.3 p^* + 0.3 \]

\[ 0.1(p^*)^2 + 0.4 p^* - 0.3 = 0 \]

Use quadratic \( a = 0.1 \quad b = 0.4 \quad c = -0.3 \)

\[ p^* = \frac{-0.4 \pm \sqrt{(0.4)^2 - 4(0.1)(-0.3)}}{2(0.1)} \]

\[ p^* \approx 0.645 \quad \text{and} \quad p^* \approx 4.645 \]