Test	Appropriate Series	Convergent	Divergent	Extras
Test for Divergence	$\sum_{n=1}^{\infty} a_n$		$\lim_{n\to\infty} a_n \neq 0$ *or DNE*	This test cannot be used to show convergence.
Geometric Series	$\sum_{n=1}^{\infty} ar^{n-1}$ or $\sum_{n=0}^{\infty} ar^n$	$\lvert r\rvert<1$	$\lvert r\rvert\geq 1$	Sum: $S = \frac{a}{1-r}$
p-series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p>1$	$p\leq 1$	
Integral ($f(x)$ cont, pos, dec)	$\sum_{n=1}^{\infty} a_n$ where $a_n=f(n)$	$\int_1^{\infty} f(x)dx$ *converges*	$\int_1^{\infty} f(x)dx$ *diverges*	Remainder: $\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_n^{\infty} f(x)dx$
Direct Comparison	$\sum_{n=1}^{\infty} a_n$ $(a_n, b_n>0)$	$a_n\leq b_n$ and $\sum_{n=1}^{\infty} b_n$ *converges*	$a_n\geq b_n$ and $\sum_{n=1}^{\infty} b_n$ *diverges*	$R_n \leq T_n \leq \int_n^{\infty} f(x)dx$
Limit Comparison	$\sum_{n=1}^{\infty} a_n$ $(a_n, b_n>0)$	$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ *converges*	$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ *diverges*	
Alternating Series	$\sum_{n=1}^{\infty} (-1)^{n-1} b_n$ or $\sum_{n=1}^{\infty} (-1)^n b_n$	*i.)* b_n decreasing *ii.)* $\lim_{n\to\infty} b_n = 0$		Remainder: $\lvert R_n\rvert \leq b_{n+1}$
Root	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = L < 1$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = L > 1$	Inconclusive if $\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = 1$
Ratio	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = L < 1$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = L > 1$	Inconclusive if $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = 1$

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1.Determine whether the **sequence** $a_n = \cos(n\pi/4)$ is increasing, decreasing, or not monotonic. Is it bounded?

2. If $\sum_{n=1}^{\infty} a_n = 5$ and $s_n = a_1 + a_2 + a_3 + ... a_n$ then,

(a) what is $\lim_{n\to\infty} s_n$?

(b) what is $\lim_{n\to\infty} a_n$?

3.True or False. Circle the correct answer. No justification needed.

T F a) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ is convergent.

T F b) If $\sum_{n=1}^{\infty} c_n 6^n$ is convergent, then $\sum_{n=1}^{\infty} c_n (-2)^n$ is convergent.

T F c) If $0 \le a_n \le b_n$ and $\sum_{n=1}^{\infty} b_n$ is convergent, then $\sum_{n=1}^{\infty} a_n$ is convergent.

T F d) $\sum_{n=1}^{\infty} 5(x)^{2n} = \frac{5x^2}{1-x^2}$

T F e) The more terms in a Taylor polynomial, the better the approximation.

4.Find the sum of the series $2 - \frac{2}{3} + \frac{2}{9} - \frac{2}{27} ...$

5.How large must N be to ensure the error in using $S_N = \sum_{n=1}^{N} \frac{1}{n^{3/2}}$ to approximate the sum, $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ with error less than 0.01?

6. Choose any two of the three. **Clearly cross out the one that you DO NOT want graded.**
Determine if the following series are absolutely convergent, conditionally convergent or divergent. Justify your answer, including stating the test and checking that the series satisfies the conditions for the test.

(a) $\sum_{n=1}^{\infty} \frac{2^{n+1}}{(n+1)!}$

(b) $\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$

(c) $\sum_{n=1}^{\infty} \left(\frac{2n+1}{3n-2} \right)^n$

7.Given that the geometric series $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ for |x|<1, find a power series representation for $\int \frac{2}{1-x^4} dx.$

8.Determine the **radius** and **interval** of convergence of the series $\sum_{n=0}^{\infty} \frac{2^n x^n}{n+3}$.

9. Find the Taylor polynomial, $T_3(x)$, for $f(x) = e^{2x}$ about $x = 0$.

10. Use Taylor's Theorem to determine the error if the polynomial found in #9 is used to approximate $f(-0.1) = e^{2(-0.1)}$.