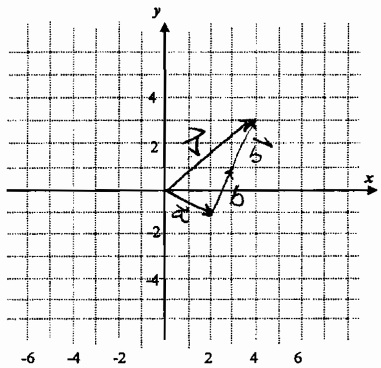
Name:____________________ Sec:__________

Math 1297 – Spring 2008 - Exam 1-10 points each unless noted otherwise.

1. Given the vectors $\vec{a} = \langle 2, -1 \rangle$ *and* $\vec{b} = \langle 1, 2 \rangle$, find $\vec{v} = \vec{a} + 2\vec{b}$. Sketch $\vec{a}, \vec{b}$ *and* $\vec{v}$.



$\vec{v} = \langle 2, -1 \rangle + 2\langle 1, 2 \rangle$

$\vec{v} = \langle 2, -1 \rangle + \langle 2, 4 \rangle$

$\vec{v} = \langle 4, 3 \rangle$ ✓ +4

+2 for each sketch.

2. Find the distance from the center of the sphere $x^2 + y^2 + z^2 - 2x - 4y - 8z = 4$ to the yz plane.

$x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 8z + 16 = 4 + 1 + 4 + 16$

$(x-1)^2 + (y-2)^2 + (z-4)^2 = 25$

center $(1, 2, 4)$ ✓+5

yz plane $(0, y, z)$ → closest point $(0, 2, 4)$

Distance: 1 unit ✓+5

3. The vectors $\vec{a} = \langle x, 1, -3 \rangle$ *and* $\vec{b} = \langle 6, -3, 9 \rangle$ are orthogonal when x=__5__.

$\vec{a} \cdot \vec{b} = 0$ ✓+5

$\vec{a} \cdot \vec{b} = 6x + (1)(-3) + (-3)(9) = 6x - 3 - 27 = 0$ ✓+3

$6x - 30 = 0$

$6x = 30$

$x = 5$ ✓ +2

4. Find the angle between the vectors $\vec{a} = \langle 3, -1, 2 \rangle$ *and* $\vec{b} = \langle -4, 0, 2 \rangle$.

$\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \|\vec{b}\|}$ ✓+2

$\|\vec{a}\| = \sqrt{3^2 + (-1)^2 + 2^2} = \sqrt{14}$

$\|\vec{b}\| = \sqrt{(-4)^2 + 0^2 + 2^2} = \sqrt{20}$ ✓+3

$\cos\theta = \dfrac{(3)(-4) + (-1)(0) + (2)(2)}{\sqrt{14} \cdot \sqrt{20}}$ ✓+2

$\theta = \cos^{-1}\left(\dfrac{-8}{\sqrt{14}\sqrt{20}}\right) \approx 2.069$ radians ✓+3

5. Find a unit vector that is orthogonal to both $\vec{a} = \vec{i} - 4\vec{j} + \vec{k}$ *and* $\vec{b} = 2\vec{i} + 3\vec{j}$.

$$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 1 & -4 & 1 \\ 2 & 3 & 0 \end{vmatrix} = \vec{i}\begin{vmatrix} -4 & 1 \\ 3 & 0 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix} + k\begin{vmatrix} 1 & -4 \\ 2 & 3 \end{vmatrix}$$

$\checkmark +2$

$$= \vec{i}(0-3) - j(0-2) + k(3 - {-8}) = -3\vec{i} + 2\vec{j} + 11\vec{k} \quad \checkmark +3$$

$$\|\vec{a}\times\vec{b}\| = \sqrt{(-3)^2 + (2)^2 + (11)^2} = \sqrt{134} \quad \checkmark +2$$

$$\frac{\vec{a}\times\vec{b}}{\|\vec{a}\times\vec{b}\|} = \frac{1}{\sqrt{134}}\langle -3, 2, 11\rangle = \left\langle \frac{-3}{\sqrt{134}}, \frac{2}{\sqrt{134}}, \frac{11}{\sqrt{134}} \right\rangle \quad \checkmark +3$$

6. Find the symmetric equation of the line that passes through the point (3,-2,4) and is perpendicular to the plane 2x-y+z=14.

$n = \langle 2, -1, 1\rangle$ = direction vector of the line $\checkmark +5$

$$\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z-4}{1} \quad \checkmark +5$$

7. Compute the derivative of $y = (x^2 + e^x)^x$

no log. diff. −7
no product rule −3

$$\ln y = \ln (x^2 + e^x)^x$$

$$\ln y = x \ln (x^2 + e^x) \quad \checkmark +3$$

$$\frac{1}{y}\frac{dy}{dx} = x \cdot \frac{2x + e^x}{x^2 + e^x} + \ln(x^2 + e^x) \quad \checkmark +5$$

$$\frac{dy}{dx} = \left[\frac{x(2x+e^x)}{x^2+e^x} + \ln(x^2+e^x)\right] y$$

$$\frac{dy}{dx} = \left[\frac{x(2x+e^x)}{x^2+e^x} + \ln(x^2+e^x)\right](x^2+e^x)^x \quad \checkmark +2$$

8. Evaluate the integral. $\int \frac{e^{2x}}{1+e^{2x}}\,dx = \int \frac{\frac{1}{2}du}{u} = \frac{1}{2}\int \frac{1}{u}\,du$ +2

$u = 1+e^{2x}$

$\frac{du}{dx} = 2e^{2x}$

$\frac{1}{2}du = e^{2x}\,dx$ +3

$= \frac{1}{2}\ln|u| + C$ +3

$= \frac{1}{2}\ln|1+e^{2x}| + C$ +2

Since $1+e^{2x} > 0$ for all x

$= \frac{1}{2}\ln(1+e^{2x}) + C$

9. Given $f(x) = \sqrt{x-4}$. Compute $(f^{-1})'(2)$ without computing the inverse.

$(f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))}$ +2 $= \frac{1}{f'(8)} = \frac{1}{\frac{1}{2\sqrt{8-4}}}$ +2 $= \frac{1}{\frac{1}{2\cdot 2}} = \frac{1}{\frac{1}{4}} = \boxed{4}$ +1

$f^{-1}(2) = \square$

$f(\square) = 2$

$\sqrt{x-4} = 2$

$x - 4 = 2^2$

$x = 8$

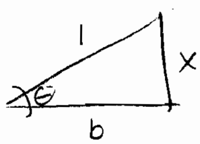
So, $f^{-1}(2) = 8$ +3

$f'(x) = \frac{1}{2}(x-4)^{-\frac{1}{2}} = \frac{1}{2\sqrt{x-4}}$ +2

10. Evaluate exactly $\cancel{\tan\left(\sin^{-1}\left(\frac{-1}{2}\right)\right)}$. $\sec(\sin^{-1}(x)) = \sec(\theta)$

$\theta = \sin^{-1}(x)$

$\sin\theta = x$ +2



+2

$1^2 = b^2 + x^2$

$b^2 = 1 - x^2$

$b = \sqrt{1-x^2}$

$= \frac{1}{\cos(\theta)}$ +2 $= \frac{1}{\frac{\sqrt{1-x^2}}{1}}$ +1

$= \frac{1}{\sqrt{1-x^2}}$ +1

for $|x| < 1$ +2