Math 1297 – Spring 2008 - Exam 2 Name: Key Sec:________

1. Compute the derivative of $y=\sinh^{-1}(e^{2x})$.

$$y' = \frac{1}{\sqrt{1+(e^{2x})^2}} \cdot 2e^{2x} \quad +5$$
$$= \frac{2e^{2x}}{\sqrt{1+e^{4x}}}$$

+5

2. $\int \frac{\sinh x}{\sqrt{\cosh x}}\,dx = \int \frac{1}{\sqrt{u}}\cdot du = \int u^{-\frac{1}{2}}\,du = 2u^{\frac{1}{2}} + C \quad +2$

+2 +2

$= 2\sqrt{\cosh x} + C \quad +2$

Let $u=\cosh x$

$\frac{du}{dx} = \sinh x \quad +3$

3. $\int \frac{1}{x^2+2x+10}\,dx = \int \underbrace{\frac{1}{x^2+2x+1-1+10}}_{+5}\,dx = \int \frac{1}{(x+1)^2+9}\,dx = \frac{1}{3}\tan^{-1}\left(\frac{x+1}{3}\right)+C$

+2 +3

4. $\int \frac{x+x^3}{x^4+2}\,dx = \int \frac{x}{x^4+2} + \frac{x^3}{x^4+2}\,dx \quad +3$

$= \int \frac{x}{(x^2)^2+2}\,dx + \int \frac{x^3}{x^4+2}\,dx = \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x^2}{\sqrt{2}}\right) + \frac{1}{4}\ln|x^4+2| + C$

+1

let $u=x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\int \frac{\frac{1}{2}du}{u^2+2} = \frac{1}{2}\int \frac{1}{u^2+2}\,du$

$= \frac{1}{2}\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right)$

+3

Let $u = x^4+2$

$du = 4x^3\,dx$

$\frac{1}{4}du = x^3\,dx$

$\int \frac{\frac{1}{4}du}{u} = \frac{1}{4}\int \frac{1}{u}\,du$

$= \frac{1}{4}\ln|u| \quad +3$

5. $\int \sin^5 x dx = \int \sin^4 x \sin x \, dx$ ✓+3 -5 table

let $u=\cos x$, $-du = \sin x dx$

$= \int (1-\cos^2 x)^2 \sin x \, dx$ ✓+2

$= \int (1-u^2)^2(-du)$ ✓+1

$= -\int (1-u^2)(1-u^2)\,du$

$= -\int (1-2u^2+u^4)\,du$ ✓+1

$= -\left(u - \frac{2}{3}u^3 + \frac{1}{5}u^5\right) + C$ ✓+2

$$\boxed{= -\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C}$$ ✓+1

6. $\int \frac{\sqrt{x^2-9}}{x} dx$ using the identity $\tan^2\theta = \sec^2\theta - 1$.

let $x = 3\sec\theta$

$dx = 3\sec\theta\tan\theta\,d\theta$ ✓+2

$= \int \frac{\sqrt{(3\sec\theta)^2-9}}{\cancel{3\sec\theta}} \cdot \cancel{3\sec\theta}\tan\theta\,d\theta$ +2 $= \int \sqrt{9(\sec^2\theta-1)}\tan\theta\,d\theta$

$= \int 3\tan\theta\tan\theta\,d\theta = 3\int \tan^2\theta\,d\theta$ ✓+2

$= 3\int(\sec^2\theta - 1)\,d\theta = 3\tan\theta - 3\theta + C$ ✓+2   ok to use table to get here

$= 3\left(\frac{\sqrt{x^2-9}}{3}\right) - 3\sec^{-1}\left(\frac{x}{3}\right) + C$ ✓+2

$$\boxed{= \sqrt{x^2-9} - 3\sec^{-1}\left(\frac{x}{3}\right) + C}$$

no dx : -3

7. $\int \frac{x^2+4}{x^2+2x+1} dx$

$x^2+2x+1 \,\overline{)\, x^2 + 0x + 4}$ quotient $1 + \frac{-2x+3}{x^2+2x+1}$

$-(x^2+2x+1)$

$-2x+3$ ✓+2

$= 1 + \frac{-2x+3}{(x+1)^2} = 1 + \frac{A}{(x+1)} + \frac{B}{(x+1)^2}$ ✓+2

$\frac{-2x+3}{(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2} \Rightarrow -2x+3 = A(x+1) + B$

let $x=-1$

$-2(-1)+3 = A(0) + B \Rightarrow B = 5$ ✓+2

$A = -2$

$\int \frac{x^2+4}{x^2+2x+1} dx = \int 1 + \frac{-2}{x+1} + \frac{5}{(x+1)^2} dx$

$$\boxed{= x - 2\ln|x+1| - \frac{5}{x+1} + C}$$

+1   +1   +1

side $\int \frac{5}{(x+1)^2}dx = 5\int \frac{1}{u^2}du$

let $u = x+1$, $du = dx$ $5\int u^{-2}du$

$-5u^{-1} + c$

no long division : -3

do: $\int\left(1 - \frac{2x+3}{x^2+2x+1}\right)dx$ correctly: $x - 2\ln|x+1| + \frac{1}{x+1} +$ -2

wrong: $x - 2\ln|x+1| - \frac{1}{x+1} + C$ -3

8. $\int \frac{\sqrt{x}}{\sqrt{x}-1}dx = \int \frac{u}{u-1}\cdot 2u\,du = 2\int \frac{u^2}{u-1}\,du = 2\int u+1+\frac{1}{u-1}\,du$ +1 +1

let $u=\sqrt{x}$

$u^2 = x$

$2u\,du = dx$ +2

$$\begin{array}{r} u+1+\frac{1}{u-1} \\ u-1\,\overline{)\,u^2+0u+0} \\ -(u^2 \pm u) \\ \hline u+0 \\ -(u \pm 1) \\ \hline 1 \end{array}$$ +2

$= 2\left[\frac{1}{2}u^2 + u + \ln|u-1|\right] + C$ +2

$= u^2 + 2u + 2\ln|u-1| + C$

$= (\sqrt{x})^2 + 2\sqrt{x} + 2\ln|\sqrt{x}-1| + C$ +2

$= x + 2\sqrt{x} + 2\ln|\sqrt{x}-1| + C$

9. $\int x\sin^{-1}(x^2)dx = \int \sin^{-1}(u)\frac{1}{2}\,du$ +2 $\overset{\#93}{=} \frac{1}{2}\left[u\sin^{-1}u + \sqrt{1-u^2}\right] + C$ +3

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$ +3

$= \frac{1}{2}x^2\sin^{-1}(x^2) + \frac{1}{2}\sqrt{1-(x^2)^2} + C$ +2

$= \frac{1}{2}x^2\sin^{-1}(x^2) + \frac{1}{2}\sqrt{1-x^4} + C$

10. Using L´Hôpital's Rule, evaluate $\lim_{x\to\infty}\frac{x}{(\ln x)^2} \overset{LH}{\underset{\infty}{\overset{\infty}{=}}} \lim_{x\to\infty}\frac{1}{2(\ln x)\cdot\frac{1}{x}}$ +3 $= \lim_{x\to\infty}\frac{x}{2\ln x} \overset{LH}{\underset{\infty}{\overset{\infty}{=}}}$ +2 $\lim_{x\to\infty} 2\frac{1}{\frac{1}{x}}$ +3 $= \lim_{x\to\infty}\frac{x}{2} = [$

*alternative solution to #8.

let $u = \sqrt{x}-1$

$\sqrt{x} = u+1$

$x = (u+1)^2$

$dx = 2(u+1)du$

$\int \frac{\sqrt{x}}{\sqrt{x}-1}dx = \int \frac{u+1}{u}2(u+1)du = 2\int \frac{u^2+2u+1}{u}du = 2\int u+2+\frac{1}{u}\,du$

$= 2\left[\frac{1}{2}u^2 + 2u + \ln|u|\right] + C = u^2 + 4u + 2\ln|u| + C$

$= (\sqrt{x}-1)^2 + 4(\sqrt{x}-1) + 2\ln|\sqrt{x}-1| + C$

now the question is how to show the solutions are equal.

Clearly $2\ln(\sqrt{x}-1) = 2\ln(\sqrt{x}-1)$.

$(\sqrt{x}-1)^2 + 4(\sqrt{x}-1) = (\sqrt{x}-1)(\sqrt{x}-1) + 4\sqrt{x} - 4$

$x - 2\sqrt{x} + 1 + 4\sqrt{x} - 4$

$x + 2\sqrt{x} - 3$ constant.