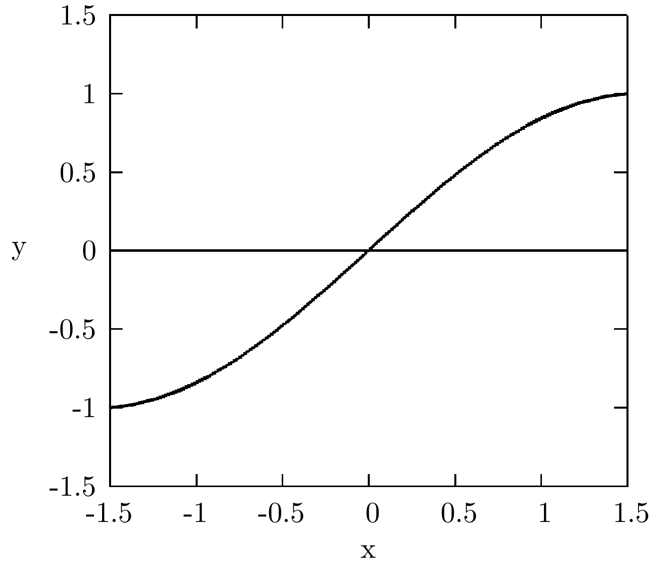

Unless otherwise noted, each part of each problem is worth 6 points.

1. Solve exactly (without calculator) for x: $\log_5(e^x) = -1$. Do not simplify.

2. Find the derivatives of the following functions. Show all work. You need not simplify your answers.

   (a) (4pts) $e^{x\ln(2)}$

   (b) $\arctan(\ln(x^2))$

   (c) (5 pts) $(\sin(x))^x$ Assume $\pi < x < 0$. (Hint: Use logarithmic differentiation.)

3. (4pts) Let $\vec{a}$ and $\vec{b}$ be placed in the plane so that the tail of $\vec{b}$ is at the tip of $\vec{a}$. Write an expression in terms of $\vec{a}$ and $\vec{b}$ for the vector from the tip of $\vec{b}$ to the tail of $\vec{a}$. Hint: draw a picture.

4. Let $P = (1, -3, 5)$ and $Q = (1, 1, 1)$.

   (a) (4pts) What is the length of $\vec{PQ}$? Give an exact answer, not a calculator approximation.

   (b) (4pts) Write any vector parametric equation for the line through P and Q.

5. (4pts) Circle ALL of the following equations that define surfaces which intersect the xy plane in a circle?

   A. $x^2 + y^2 + z^2 = 1$ B. $x^2 - y^2 + z^2 = 1$ C. $x^2 + y^2 - z^2 = 1$ D. $x^2 - y^2 - z^2 = 1$

6. (4pts) The graph of a function is given in the accompanying figure. Sketch the graph of its inverse on the same figure.

7. Assume the following: $f(2) = 1, f^{-1\prime}(1) = 3$. What does this imply about the derivative of the original function f? Suggestion: Draw a figure.

   A. $f'(2) = \frac{1}{3}$ B. $f'(2) = -\frac{1}{3}$ C. $f'(1) = \frac{1}{3}$ D. $f'(1) = -\frac{1}{3}$ E. $f'(3) = 2$

8. (4pts) Let $\mathbf{N} =< 1, 2, 5 >$. Write an equation (in any form) of the plane P containing the point $(1, 0, -3)$ with normal vector $\mathbf{N}$?

9. What is a normal vector to the plane which contains the three points: $(3, -1, 2), (2, 2, 2)$, and $(0, 5, 3)$

10. (3 pts each) True-False. Write TRUE or FALSE answers to the left of each problem. No justification necessary. No partial credit.

   (a) If x and y are both positive, $\ln(x + y) = \ln(x) + \ln(y)$

   (b) $\cos^{-1}(x) = \dfrac{1}{\cos(x)}$.

   (c) $\lim_{x\to\infty} e^{-x^2} = \infty$.

   (d) $\dfrac{d}{dx}(x^{\pi}) = \pi x^{\pi-1}$

   (e) $\dfrac{d}{dx}(\cosh(x)) = \sinh(x)$

   (f) The vector projection of $1\vec{i} + 2\vec{j} + 3\vec{k}$ onto the vector $2\vec{k}$ is $3\vec{k}$.

11. Evaluate $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$

12. Show $\frac{d}{dx}a^x = a^x \ln(a)$, assuming you know that $\frac{d}{dx}e^x = e^x$.

13. Show $\dfrac{d}{dx}(\ln x) = \dfrac{1}{x}$ (for $x > 0$) using the fact that $\ln x$ is the inverse of the e^x and the derivative of e^x is e^x.

14. Let $\vec{a} =< a_1, a_2, a_3 >$, and $\vec{b} =< b_1, b_2, b_3 >$. Show that the vector $\vec{a} \times \vec{b}$ is perpendicular to $\vec{b}$.

Formulas:

Inverse trig function derivatives:

- $\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}(\cos^{-1}(x)) = -\frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$
- $\frac{d}{dx}(\csc^{-1}(x)) = -\frac{1}{x\sqrt{x^2-1}}$
- $\frac{d}{dx}(\sec^{-1}(x)) = \frac{1}{x\sqrt{x^2-1}}$
- $\frac{d}{dx}(\cot^{-1}(x)) = -\frac{1}{1+x^2}$

Hyperbolic function derivatives:

- $\frac{d}{dx}(\sinh(x)) = \cosh(x)$
- $\frac{d}{dx}(\cosh(x)) = \sinh(x)$
- $\frac{d}{dx}(\tanh(x)) = \operatorname{sech}^2(x)$
- $\frac{d}{dx}(\operatorname{csch}(x)) = -\operatorname{csch}(x)\coth(x)$
- $\frac{d}{dx}(\operatorname{sech}(x)) = -\operatorname{sech}(x)\tanh(x)$
- $\frac{d}{dx}(\coth(x)) = -\operatorname{csch}^2(x)$

Inverse hyperbolic function derivatives:

- $\frac{d}{dx}(\sinh^{-1}(x)) = \frac{1}{\sqrt{1+x^2}}$
- $\frac{d}{dx}(\cosh^{-1}(x)) = \frac{1}{\sqrt{x^2-1}}$
- $\frac{d}{dx}(\tanh^{-1}(x)) = \frac{1}{1-x^2}$
- $\frac{d}{dx}(\operatorname{csch}^{-1}(x)) = -\frac{1}{|x|\sqrt{x^2+1}}$
- $\frac{d}{dx}(\operatorname{sech}^{-1}(x)) = -\frac{1}{x\sqrt{x^2-1}}$
- $\frac{d}{dx}(\coth^{-1}(x)) = \frac{1}{1-x^2}$

Projections:

1. $\text{comp}_{\vec{a}}\vec{b} = \vec{b} \cdot \frac{\vec{a}}{|\vec{a}|}$
2. $\overrightarrow{\text{proj}}_{\vec{a}}\vec{b} = (\vec{b} \cdot \frac{\vec{a}}{|\vec{a}|})\frac{\vec{a}}{|\vec{a}|}$