Math 1297, Calculus II

Lecture Section 8

Proofs (and hints) to know for Test 1

1. Show $\vec{a} \times \vec{b}$ is perpendicular to $\vec{a}$. (Hint: Dot $\vec{a} \times \vec{b}$ with $\vec{a}$ and show it equals zero. See p. 852.)

2. Show the inverse derivative formula (7.1): $f^{-1'}(x) = \frac{1}{f'(f^{-1}(x))}$. (Hint: Don't use the book's technique on p. 418. Instead, start with $f(f^{-1}(x)) = x$ and take the derivative of both sides, using the chain rule on the left side: $f'(f^{-1}(x))f^{-1'}(x) = 1$. Solve for the term $f^{-1'}(x)$.)

3. Show $\frac{d}{dx}\ln(x) = \frac{1}{x}$, assuming you know that $\frac{d}{dx}e^x = e^x$. (Hint: Special case of 3. Start with $e^{\ln(x)} = x$, differentiate, simplify. See p. 441.)

4. Show $\frac{d}{dx}\arctan(x) = \frac{1}{1+x^2}$. (Hint: Special case of 3. Start with $\tan(\arctan(x)) = x$, differentiate, simplify using the appropriate triangle..)

5. Show $\log_a(x) = \frac{\ln(x)}{\ln(a)}$. (Hint: $y = \log_a(x) \iff a^y = x$. Now take ln of both sides.)

6. Show $\frac{d}{dx}a^x = a^x \ln(a)$, assuming you know that $\frac{d}{dx}e^x = e^x$. (Hint: Rewrite a^x as $(e^{\ln(a)})^x = e^{x\ln(a)}$. Differentiate.)

7. Show $\frac{d}{dx}\cosh(x) = \sinh(x)$ (or vice versa). (Hint: Direct computation using the definition of $\sinh(x)$ and $\cosh(x)$.)

8. Show $\int \tan(x)dx = \ln|\sec(x)| + C$. (Hint: Write $\tan(x)$ as $\frac{\sin(x)}{\cos(x)}$ and integrate with a "u-substitution": $u = \cos(x)$).