# Math 1297, Calculus II 

Test 2 answers as of May 4, 2007

1. $u=\cos (x)$
2. $4^{5} \int \sin ^{3}(\theta) \cos ^{2}(\theta)$
3. $u=3 x, d v=\sin (5 x)$
4. (a) $\frac{A}{x-3}+\frac{B}{(x-3)^{2}}+\frac{C}{x+1}$
(b) $\frac{A}{x}+\frac{B}{x^{2}}+\frac{C x}{x^{2}+4}+\frac{D}{x^{2}+4}$
5. $3 \ln |x|+\frac{2}{x}+\frac{1}{3} \ln |3 x-1|+C$
6. $-\frac{1}{5} \frac{x}{\left(9 x^{2}-5\right)^{\frac{1}{2}}}+C$
7. (a) 0
(b) 1
8. $R_{3}<T_{3}<M_{3}<L_{3}$
9. $\int_{1}^{e} \sqrt{1+\left(\frac{1}{x}\right)^{2}} d x$
10. $S=\int_{0}^{\pi} 2 \pi(\sin (x)+2) \sqrt{1+\cos ^{2}(x)} d x$
11. Given $\epsilon>0$ there exists an integer $N$ such that $n>N$ implies $\left|a_{n}-L\right|<\epsilon$.
12. (a) $2,2 / 3,2 / 9$
(b) 0
13. (a) $3 / 5$ (divide numerator and denominator by $n^{2}$ and then take the limit as $n$ goes to $\infty$ )
(b) 0 Use the sandwich theorem: $-\frac{1}{n} \leq \frac{\cos (n)}{n} \leq \frac{1}{n}$. The two outside sequences go to zero, so the middle one must also go to zero.
14. False. A counter example is the harmonic series.
15. (a) i geometric, ii NO since $r=3 / 5>1$.
(b) i Not geometric, ii No since $a_{n} \rightarrow 1 / 3 \neq 0$
(c) i Not geometric, ii NO since it is the harmonic series which is known to diverge
16. Let $s_{n}=2+\frac{2}{5}+\ldots+\frac{2}{5^{n-1}}$. Then $\frac{1}{5} s_{n}=\frac{2}{5}+\ldots+\frac{2}{5^{n}}$. Subtracting these two equations gives $s_{n}-\frac{1}{5} s_{n}=2-\frac{2}{5^{n}}$. Solve for $s_{n}$ to get $s_{n}=\frac{2-\frac{2}{5^{n}}}{\left(1-\frac{1}{5}\right)} \rightarrow \frac{2}{\left(1-\frac{1}{5}\right)}=\frac{5}{2}$
17. Extra Credit. $2 \ln \left|x^{2}-2 x+2\right|+5 \arctan (x-1)+C$
