Practice problems for the Final Exam from Sections 15.2.

1. Let $f(x,y) = \dfrac{2x^2+3y^2}{x^2+y^2}$. Compute $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y)$ **along the following paths**:

   (a) the positive x-axis

   (b) the negative y-axis

   (c) along any line $y = mx$

   What can you conclude about $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y)$?

2. Let $f(x,y) = \dfrac{x^2y}{x^4+y^2}$. Compute $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y)$ along the following paths:

   (a) along any line $y = mx$

   (b) along the parabolas $y = cx^2$

   What can you conclude about $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y)$?

Answers:

1. (a) 2 (In fact, $f(x,0) = 2$ for any x.)

   (b) 3

   (c) $\frac{2+3m^2}{1+m^2}$

   The limit does not exist. (As soon as limits along any two paths are not equal, the limit cannot exist. So parts a and b are enough to conclude the limit does not exist.)

2. (a) zero, no matter what m is (Replace y with mx, divide numerator and denominator by x^2, and take the limit as $x \to 0$.)

   (b) $\frac{c}{1+c^2}$ (Replace y with cx^2, cancel x^4. No x's remain, so f is "constant along parabolas.")

   The limit does not exist (even though the limit along any straight line is zero!!!)