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Diff. Equations and Lin. Alg.
Math 3280
EC Quiz 11, Spring 2020
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1. Consider the differential equation $y''(t) - 4y'(t) + 5y(t) = 0$.

   (a) (2pts) Solve using "guess e^{rt}." $y = e^{rt} \Rightarrow y' = re^{rt},\ y'' = r^2 e^{rt}$

   Plug in: $r^2 e^{rt} - 4(re^{rt}) + 5(e^{rt})$

   $= (r^2 - 4r + 5)e^{rt} = 0$

   if $r^2 - 4r + 5 = 0 \Rightarrow r = \frac{4 \pm \sqrt{16-20}}{2} = 2 \pm i \Rightarrow y(t) = c_1 e^{2t}\cos t + c_2 e^{2t}\sin t.$

   (b) (2pts) Solve using Laplace transforms. Use $y(0) = y_0$, and $y'(0) = y_1$. Stop when you find the transform of the solution, $Y(s)$. Do not take the inverse transform to find $y(t)$.

   Transform: $(s^2 Y(s) - s y_0 - y_1) - 4(s Y(s) - y_0) + 5Y(s) = 0$

   Solve for $Y(s)$: $Y(s)(s^2 - 4s + 5) = s y_0 + y_1 - 4y_0$

   $Y(s) = \frac{s y_0 + y_1 - 4y_0}{s^2 - 4s + 5}$

   (c) (3pts) Solve by converting to a system and writing in the form $\vec{x}'(t) = A\vec{x}$. Stop once you find the eigenvalues for A. You need not find any eigenvectors or the solution.

   let $x_1 = y$, $x_2 = y'$. Then $x_1' = x_2$

   $x_2' = y'' = 4y' - 5y = 4x_2 - 5x_1$

   i.e., $x_1' = 0x_1 + 1x_2$, $x_2' = -5x_1 + 4x_2$ $\Rightarrow \vec{x}' = \begin{pmatrix} 0 & 1 \\ -5 & 4 \end{pmatrix}\vec{x}$

   $A = \begin{bmatrix} 0 & 1 \\ -5 & 4 \end{bmatrix}$

   $\det(A - \lambda I) = \begin{vmatrix} 0-\lambda & 1 \\ -5 & 4-\lambda \end{vmatrix}$

   $= (-\lambda)(4-\lambda) + 5$

   $= \lambda^2 - 4\lambda + 5$

   $= 0$ if $\lambda = \frac{4 \pm \sqrt{16-20}}{2}$

   $= 2 \pm i$

2. (3pts) Show that if $S = \text{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \right\}$

   then S is a vector subspace of $\Re^4$.

i) Let $\vec{x}, \vec{y} \in S$. $\Rightarrow \vec{x} = c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 4 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ and $\vec{y} = k_1 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 4 \end{bmatrix} + k_2 \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

$\Rightarrow \vec{x} + \vec{y} = \left(c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 4 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right) + \left(k_1 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 4 \end{bmatrix} + k_2 \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right) = (c_1 + k_1) \begin{bmatrix} 1 \\ 1 \\ 0 \\ 4 \end{bmatrix} + (c_2 + k_2) \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \in S.$

(Since this is a linear comb of the 2 vecs)

ii) Let $\vec{x} \in S$, $c \in \mathbb{R}$

$\vec{x} = c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 4 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow c\vec{x} = (cc_1) \begin{bmatrix} 1 \\ 1 \\ 0 \\ 4 \end{bmatrix} + (cc_2) \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \in S.$

i) and ii) $\Rightarrow S$ is a subspace