Name A.K.

Diff. Equations and Lin. Alg.
Math 3280, B. Peckham
Quiz 8, Fall 2020

Directions: Print out or write answers on a separate sheet of paper. Include your name, start time, stop time, and an honor statement saying "I used no outside resources."

Scan or take a picture when done. Upload via Canvas to Quiz 8. The quiz window will be open from 9:10 to 10:00.

1. Consider the differential equation: $y'' + y' - 12y = 0$. Solve using the following steps:
   (a) (1 pt) Convert to a system of first order differential equations
   (b) (1 pts) Write your system in the form $\dot{\vec{x}} = A\vec{x}$
   (c) (4pts) Find the eigenvalues and eigenvectors of A
   (d) (2 pts) Use the eigenvalues and eigenvectors from (c) to write the general solution to your system. If you don't have an answer to part (b), write the form of the general solution.

(a) Let $v = y'$ Then $y'' + y' - 12y = 0$ becomes $v' + v - 12y = 0$

So the system is $y' = v$
$v' = 12y - v$

(b) Let $\vec{x} = \begin{bmatrix} y \\ v \end{bmatrix}$, $A = \begin{bmatrix} 0 & 1 \\ 12 & -1 \end{bmatrix}$ So in vector form, $\begin{bmatrix} y \\ v \end{bmatrix}' = \begin{bmatrix} 0 & 1 \\ 12 & -1 \end{bmatrix}\begin{bmatrix} y \\ v \end{bmatrix}$

(c) $\det(A - \lambda I) = \begin{vmatrix} -\lambda & 1 \\ 12 & -1-\lambda \end{vmatrix} = \lambda(1+\lambda) - 12 = +\lambda^2 + \lambda - 12 = (\lambda - 3)(\lambda + 4)$

So eigenvalues are $\lambda = 3, -4$

$\lambda = 3$: $(A - 3I)\vec{v} = \begin{bmatrix} -3 & 1 \\ 12 & -4 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow v_2 = t,\ -3v_1 + v_2 = 0,\ v_1 = \frac{1}{3}v_2 \Rightarrow \vec{v} = t\begin{bmatrix} 1/3 \\ 1 \end{bmatrix}$

Use $(t = 3) \Rightarrow \vec{v} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ (or $\begin{bmatrix} 1/3 \\ 1 \end{bmatrix}$)

$\lambda = -4$ $(A + 4I)\vec{v} = \begin{bmatrix} 4 & 1 \\ 12 & 3 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow v_2 = t,\ 4v_1 + v_2 = 0 \Rightarrow v = t\begin{bmatrix} -1/4 \\ 1 \end{bmatrix}$

I'll use $t = -4$: $\begin{bmatrix} 1 \\ -4 \end{bmatrix}$

(d) $\therefore \begin{bmatrix} y(t) \\ v(t) \end{bmatrix} = c_1 e^{3t}\begin{bmatrix} 1 \\ 3 \end{bmatrix} + c_2 e^{-4t}\begin{bmatrix} 1 \\ -4 \end{bmatrix}$

Write the lowest order linear differential operator that annihilates the following functions:

(a) (1 pt) $e^{2x} + e^{3x}$ $(D-2)(D-3) = D^2 - 5D + 6$

(b) (1 pt) x^3 D^4

(c) (EC 2 pts) $xe^{-x}\cos(5x)$

roots $-1 \pm 5i$, repeated $(r - (-1+5i))(r - (-1-5i)) = (r+1)^2 + 5^2$

repeated $\Rightarrow ((r+1)^2 + 5^2)^2 \Rightarrow$ annihilator is $((D+1)^2 + 5^2)^2$