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Diff. Equations and Lin. Alg.
Math 3280
Quiz 9, Spring 2020
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1. (3 pts) Show directly from the definition of Laplace transform that the Laplace transform of e^{3t} is $\frac{1}{s-3}$.

$$\mathcal{L}\{e^{3t}\}(s) = \int_0^\infty e^{-st} e^{3t}\, dt = \int_0^\infty e^{-(s-3)t}$$

$$= \frac{e^{-(s-3)t}}{-(s-3)} \Big|_0^\infty = \frac{e^{-(s-3)\infty} - 1}{-(s-3)} = \frac{0-1}{-(s-3)} = \underline{\underline{\frac{1}{s-3}}}$$

2. (4pts) Use the Laplace transform method to solve the following initial value problem: $y' = 3y, y(0) = 5$.

Transform: $sY(s) - y(0) = 3Y(s)$

Solve for $Y(s)$: $Y(s)(s-3) = y(0)$

$$Y(s) = \frac{y(0)}{s-3}$$

Inverse transform:

$y(t) = y(0)\, e^{3t}$ (from Tables)

$= \underline{\underline{5e^{3t}}}$

3. (3 pts) Transform the following differential equation using the Laplace transform. Solve the transformed equation. (That is, solve for $Y(s)$; do not "undo" the transform to find $y(t)$.) $y'' + 0y' + 16y = 0, y(0) = 2, y'(0) = 1$.

Transform: $\left(s^2 Y(s) - s\,y(0) - y'(0)\right)$

$+ 16\, Y(s) = 0$

Solve for $Y(s)$: $Y(s) \cdot (s^2+16) - s\cdot 2 - 1 = 0$

ie, $Y(s)(s^2+16) = 2s+1$

$$\underline{Y(s) = \frac{2s+1}{s^2+16}}$$

4. (2pts EC) Complete problem 3. That is, find the inverse Laplace transform of $Y(s)$ from problem 3 to find the solution to the initial value problem in problem 3. You may use the tables provided.

Rewrite $Y(s) = \frac{2s+1}{s^2+16} = 2\,\frac{s}{s^2+4^2} + \frac{1}{4}\,\frac{4}{s^2+4^2}$

Inverse transform: $y(t) = \underline{2\cos(4t) + \frac{1}{4}\sin(4t)}$ (from Tables)