Math 3280, Differential Equations with Linear Algebra
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Brief Topic Summary for Test 2

Differential Equations:

1. Analytic Solutions (Sections 5.1, 5.2)
   (a) Structure of general solution to $y' + p(x)y = q(x)$: $y(x) = y_c(x) + y_p(x) = c_1y_1(x) + y_p(x)$.
   (b) Structure of general solution to $y'' + p(x)y' + q(x)y = r(x)$: $y(x) = y_c(x) + y_p(x) = c_1y_1(x) + c_2y_2(x) + y_p(x)$.
   (c) Finding $y_1(x)$ and $y_2(x)$ for $ay'' + by' + cy = 0$ (constant coefficient linear, homogeneous - if the characteristic polynomial has real distinct or real repeated roots).
   (d) Using initial conditions, solve for c_1 and c_2 for second order linear differential equations.
   (e) Linear independence of y_1 and y_2. Use of Wronskian as a shortcut. More generally, linear independence of n functions.
2. Qualitative Solutions / Numerical Solutions (labs - not on test)
3. Models (labs - not on test)
   (a) Exponential growth (population), decay (radioactive decay)
   (b) Heating/Cooling
   (c) Falling object/parachute
   (d) Mixing
   (e) Exponential population growth/decay and logistic growth

Linear Algebra (Secs 3.1-3.6, 4.1-4.4, 4.7)

1. Solve $Ax = b$ (Row reduction, row echelon form, reduced row echelon form and interpretation of row reduced matrices for no solution, unique solution, infinity of solutions, number of free parameters, dimension and codemension of the set of solutions)
2. For A an $n \times n$ matrix: Compute Det(A), A^{-1} using row reduction; for a 2×2 matrix, know the formula for A^{-1} (assuming $\det(A) \neq 0$).
3. Det$(A) \neq 0 \Leftrightarrow A^{-1}$ exists $\Leftrightarrow$ there is a unique solution to $A\mathbf{x} = \mathbf{b}$
4. Vector Space/subspace, basis, linearly independent/dependent - including formal equations that must be satisfied to be dependent, span - including formal equations that must be satisfied for a given set of vectors to to span a (sub)space, dimension
5. Vector space examples: $\Re^2, \Re^3, \Re^n, M_{m\times n}, F$.
6. Vector subspace examples:
   (a) in $\Re^n$: origin, lines through origin, (hyper)planes through the origin, solutions to $A\mathbf{x} = \mathbf{0}$ (including proof).
   (b) in F: All polynomials; all polynomials of degree n or less (n + 1-dimensional); solutions to $y' + p(x)y = 0$ (1 dimensional) or $y'' + p(x)y' + q(x)y = 0$ (2 dimensional).
   (c) in $M_{n\times n}$: Diagonal matrices, triangular matrices, symmetric matrices, ...
   (d) in any vector space: linear combinations of any set of vectors
   (e) Proof of a subset being a subspace, especially (a) and (b) (closed under vector addition and scalar multiplication) vs. example showing a subset is not a subspace. Note that properties a-h in Sec. 4.2 are "inherited" from the larger "known" vector space.
   (f) Proof that differences of solutions to $A\mathbf{x} = \mathbf{b}$ are solutions to $A\mathbf{x} = \mathbf{0}$. Analogous statement for solutions to linear differential equations, at least first and second order.
   (g) Shortcuts if you know the dimension of a vector (sub)space is n: any set of more that n vectors must be linearly dependent; no set of fewer than n vectors can span the space. Any set of n vectors that either is linearly independent or spans the space also has the other property.