Math 3280: DE+LA. Test 3 partial answers. Prof. Bruce Peckham

1. $y(x) = c_1 e^{4x} + c_2 e^{-x}$

2. $y_p(x) = \frac{1}{2} e^{5x}$

3. $(D-2)(D+4)$ or $(D+4)(D-2)$ or $D^2 + 2D - 8$.

4. $y_p(x) = A\cos(2x) + B\sin(2x) + Cxe^{2x}$

5. proof in book, Example 1, Sec. 10.1. Replace 1 with 2.

6. $y(t) = te^{2t} + 2e^{2t}$

7. $Y(s) = \frac{3s^2+6}{(s^3+2s+1)}$

8. $f(t) = 3e^{-2t}\cos(4t) - \frac{5}{2}e^{-2t}\sin(4t)$

9. (a) Let $v = \dot{x}$. Then $\dot{v} = -2x - \frac{2}{5}v + \frac{F_0}{25}\cos(\gamma t)$.

   (b) Vector form: $\vec{x}\,' = \begin{bmatrix} 0 & 1 \\ -2 & -2/5 \end{bmatrix} \vec{x} + \begin{bmatrix} 0 \\ \frac{F_0}{25}\cos(\gamma t) \end{bmatrix}$, where $\vec{x} = \begin{bmatrix} x \\ v \end{bmatrix}$.

   (c) $c_1 e^{-\frac{1}{5}t}\cos(\frac{7}{5}t) + c_2 e^{-\frac{1}{5}t}\sin(\frac{7}{5}t)$

10. $\begin{bmatrix} 2e^{3t}\cos(4t) - 5e^{3t}\sin(4t) \\ e^{3t}\cos(4t) \end{bmatrix}$

    Hint: use $e^{(3+4i)t} = e^{3t}(\cos(4t) + i\sin(4t))$, multiply, and collect real and imaginary parts. For this test question, ignore the imaginary parts.

11. Eigenvector for or eigenvalue $1+2i$: $\begin{bmatrix} 1 \\ -i \end{bmatrix}$

    (or any (complex) multiple of it, like $\begin{bmatrix} i \\ 1 \end{bmatrix}$)

12. $\vec{x}(t) = c_1 e^{2t}\begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 e^{-t}\begin{bmatrix} 1 \\ -3 \end{bmatrix}$

    Note that you need not compute the eigenvalues from scratch since I gave you two eigenvectors. You merely need to multiply the matrix $\begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$ by each eigenvector to see which eigenvalue corresponds to which eigenvectore. Since the matrix is upper triangular, then the eigenvalues of 2 and -1 are the diagonal elements.

13. (Extra Credit) Since y_1 and y_2 are known solutions to $L[y] = 0$, then $y_1'' + 5y_1' - 3y_1 = 0$ and $y_2'' + 5y_2' - 3y_2 = 0$. Therefore, $L[c_1y_1 + c_2y_2] = (c_1y_1 + c_2y_2)'' + 5(c_1y_1 + c_2y_2)' - 3(c_1y_1 + c_2y_2) = c_1(y_1'' + 5y_1' - 3y_1) + c_2(y_1'' + 5y_1' - 3y_1) = c_1 0 + c_2 0 = 0$. That is, $c_1y_1 + c_2y_2$ is a solution to $L[y] = 0$.