Math 3280 Practice Test 3. B. Peckham. Please do not write on the provided Laplace Transform tables. Indicate clearly any places where you use either the Laplace transform tables or a calculator.

1. (6 pts) Find the general solution to $y'' - 3y' - 4y = 0$.

2. (6 pts) Find one particular solution to $y'' - 3y' - 4y = 3e^{5x}$. (You need not find the general solution.)

3. (6 pts) What differential operator annihilates $3e^{2x} + e^{-4x}$? Verify your answer.

4. (6 pts) Find the form of a particular solution to $y'' - y' - 2y = \cos(2x) + e^{2x}$. Do not include extraneous terms and do not evaluate the "undetermined coefficients."

5. (6 pts) Compute the Laplace transform of $f(t) = 2$ **directly from the definition of the Laplace transform** (not the tables).

6. (8 pts) Solve using the method of Laplace transforms: $y'(t) - 2y(t) = e^{2t},\ y(0) = 2$.

7. (6 pts) Compute the Laplace transform of the solution of the initial value problem:

$$y''' + 0y'' + 2y' + y = 0; \quad y(0) = 3, y'(0) = 0, y''(0) = 0.$$

(Find only $Y(s)$, not $y(t)$.) Write your answer as a polynomial (in s) over a polynomial. You need not simplify your answer.

8. (6 pts) Find the inverse Laplace transform of $F(s) = \dfrac{3s - 4}{s^2 + 4s + 20}$.

9. Consider the initial value problem

$$25\frac{d^2x}{dt^2} + 10\frac{dx}{dt} + 50x = F_0\cos(\gamma t), x(0) = -1, x'(0) = 0 \tag{1}$$

(a) (4pts) Write a system of first order differential equations which is equivalent to equation (1). Define any new variables you introduce.

(b) (3pts) Write your system from part (a) in vector form: $\vec{x}\,' = A\vec{x} + \vec{b}, \vec{x}(0) = \begin{bmatrix} a \\ b \end{bmatrix}$.

(c) (3pts) The general solution to (1) turns out to be:

$$x(t) = c_1 e^{\frac{-1}{5}t}\cos(\frac{7}{5}t) + c_2 e^{\frac{-1}{5}t}\sin(\frac{7}{5}t) + \frac{F_0}{100 - 96\gamma^2 + 25\gamma^4}[(2-\gamma^2)\cos(\gamma t) + \frac{2}{5}\gamma\sin(\gamma t)] \tag{2}$$

Circle the piece of the general solution which is the complementary solution (the general solution to: $25\frac{d^2x}{dt^2} + 10\frac{dx}{dt} + 50x = 0$).

10. (3pts) Compute the real part of $e^{(3+4i)t}\begin{bmatrix} 2+5i \\ 1 \end{bmatrix}$.

11. (6pts) Let $A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$. Find any one nonzero eigenvector for A. You may use the fact that one eigenvalue of A turns out to be $1+2i$.

12. (6 pts) Find the general solution to $\vec{x}\,' = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}\vec{x}$. You may use the fact that $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -3 \end{bmatrix}$ are both eigenvectors for $\begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$.

13. Extra Credit (6pts). Define the differential operator L by $L = D^2 + 5D - 3$, where D is the derivative operator $\frac{d}{dx}$. Assume that $y_1(x)$ and $y_2(x)$ are known functions that are solutions to $L[y] = 0$. Show that any function of the form $c_1y_1(x) + c_2y_2(x)$ (where c_1 and c_2 are constants) is also a solution to $L[y] = 0$.