Dynamical Systems
Math 5260
Lab Tasks:
Tangent (saddle-node) and period-doubling Bifurcations for $x^2 + c$

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September 13, 2019

As c decreases in the quadratic family $x^2 + c$, periodic orbits of all periods are "born" through either saddle-node bifurcations or period doubling bifurcations.

Do the following for period-q orbits for each of $q = 1, 2, ..., 4$:

By plotting the qth iterate of the quadratic family maps $x^2 + c$ for c values decreasing from 1 to -2, determine approximate (to about 3 decimal places) parameter values where fixed points for the qth iterate are born. Assume all periodic orbits are born in either tangent or period-doubling bifurcations. Determine whether each birth is due to a tangent or a period-doubling bifurcation. Explain briefly how you obtained your answers.

Suggestion: Use software to graph various iterates of $x^2 + c$ and use a slider to vary c. Desmos is good software to use. You can also use Nonlinear Web (BU Website), Mathematica, or any other plotting software. High precision parameter specification, and zooming in on the graphs enables a more accurate estimate of the bifurcation parameters. Plotting multiple iterates of $x^2 + c$ simultaneously can be useful. It can help, for example, to plot both the first and second iterates to see which fixed points of the second iterate are prime period 1 and which are prime period 2.

(In Mathematica, try Plot[{f[x,c], f[f[x,c],c]},{x,-2,2}] where f[x_,c_] :=x^2+ c and c is a fixed parameter value.)