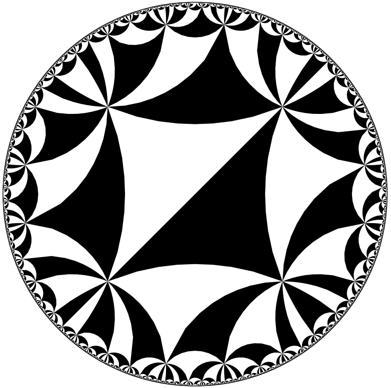
AMS Sectional Meeting, Richmond VA
Special Session on Mathematics and the Arts

Hyperbolic Truchet Tilings: First Steps

Douglas Dunham
University of Minnesota Duluth
Duluth, Minnesota
USA



Outline

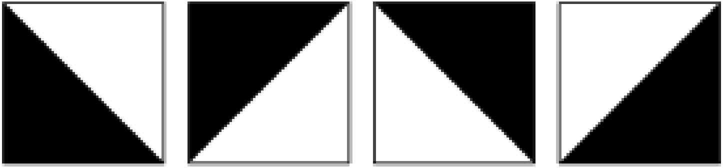
- A brief biography of Truchet
- Examples of Truchet tilings
- Hyperbolic geometry and regular tessellations
- Regular hyperbolic Truchet tilings
- Random hyperbolic Truchet tilings
- Hyperbolic circular arc Truchet tilings
- Future research

Brief Biography of Truchet

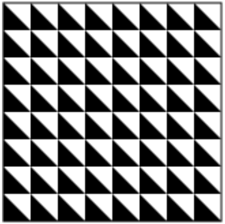
- Sébastian Truchet was born in Lyon, France in 1657.
- Interests: mathematics, hydraulics, graphics, and typography.
- Also invented sundials, weapons, and methods for transporting large trees within the Versailles gardens.
- In 1704 he invented Truchet tiles.
- Died February 5, 1729.

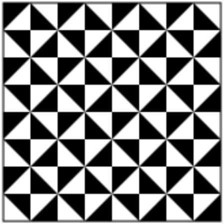
Examples of Truchet Tilings

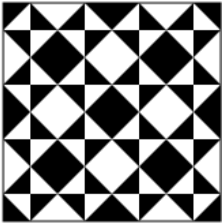
- Truchet triangle tilings
- Based on a square divided in two into a black and white triangle — 4 orientations.
- Either repeating patterns or random patterns.



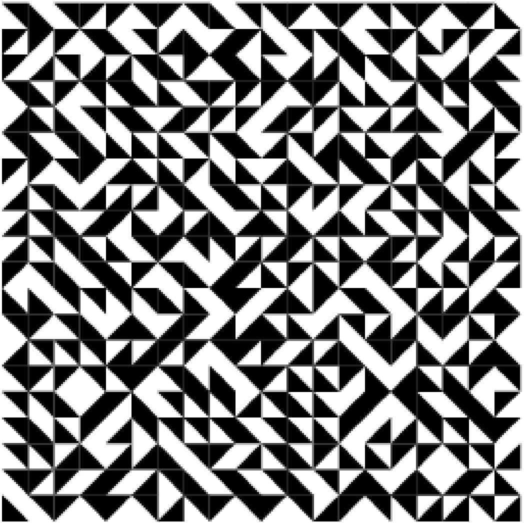
Regular Truchet Tilings







A Random Truchet Tiling



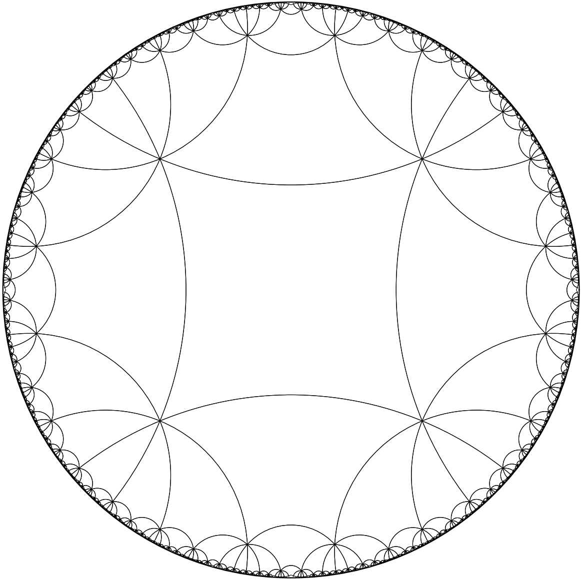
Hyperbolic Geometry and Regular Tessellations

- In 1901, David Hilbert proved that, unlike the sphere, there was no isometric (distance-preserving) embedding of the hyperbolic plane into ordinary Euclidean 3-space.
- Thus we must use *models* of hyperbolic geometry in which Euclidean objects have hyperbolic meaning, and which must distort distance.
- One such model is the *Poincaré disk model*. The hyperbolic points in this model are represented by interior point of a Euclidean circle — the *bounding circle*. The hyperbolic lines are represented by (internal) circular arcs that are perpendicular to the bounding circle (with diameters as special cases).
- This model is appealing to artests since (1) angles have their Euclidean measure (i.e. it is conformal), so that motifs of a repeating pattern retain their approximate shape as they get smaller toward the edge of the bounding circle, and (2) it can display an entire pattern in a finite area.

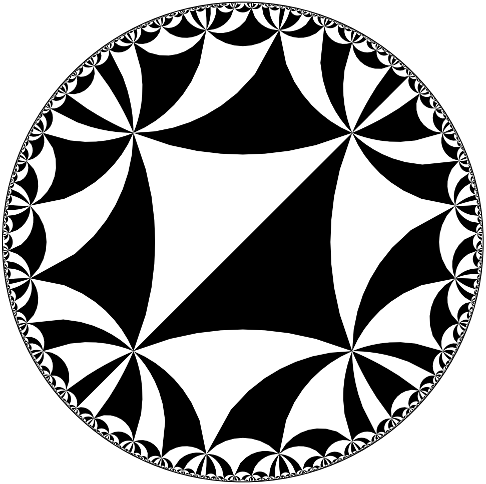
Repeating Patterns and Regular Tessellations

- A *repeating pattern* in any of the 3 "classical geometries" (Euclidean, spherical, and hyperbolic geometry) is composed of congruent copies of a basic subpattern or *motif*.
- The *regular tessellation*, $\{p, q\}$, is an important kind of repeating pattern composed of regular p-sided polygons meeting q at a vertex.
- If $(p-2)(q-2) < 4$, $\{p, q\}$ is a spherical tessellation (assuming $p > 2$ and $q > 2$ to avoid special cases).
- If $(p-2)(q-2) = 4$, $\{p, q\}$ is a Euclidean tessellation.
- If $(p-2)(q-2) > 4$, $\{p, q\}$ is a hyperbolic tessellation. The next slide shows the $\{6, 4\}$ tessellation.
- Escher based his 4 "Circle Limit" patterns, and many of his spherical and Euclidean patterns on regular tessellations.

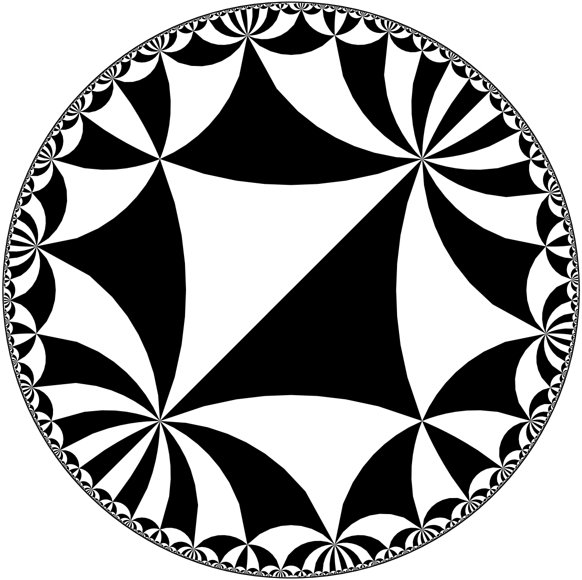
The Regular Tessellation $\{4, 8\}$ Underlying the Title Slide Image



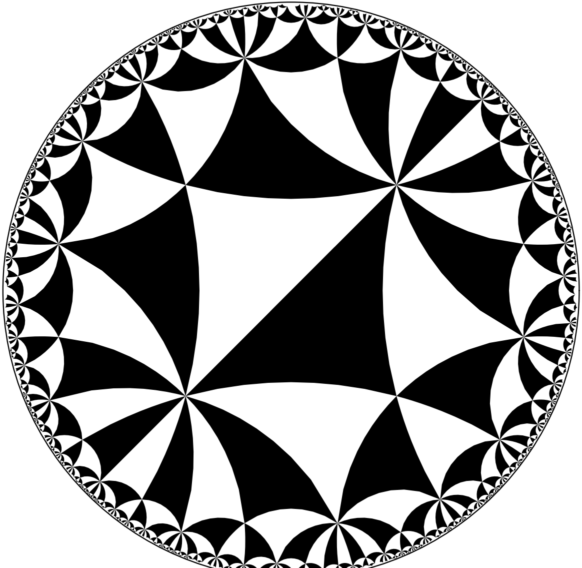
Regular Hyperbolic Truchet Tilings
The Title Slide (based $\{4, 8\}$)



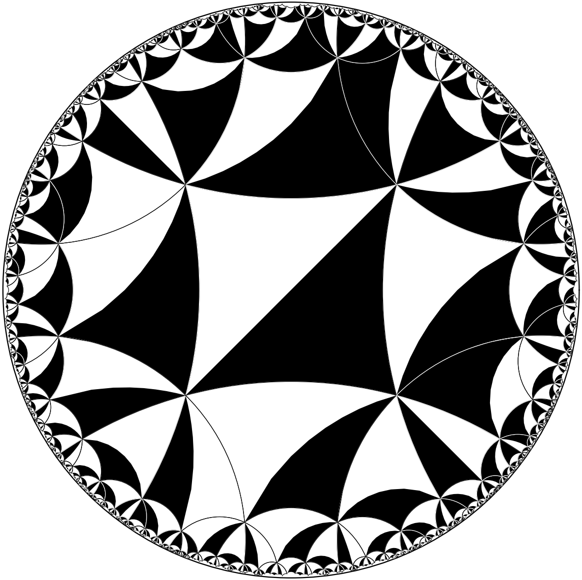
Another Regular Hyperbolic Truchet Tiling
(based on the $\{4, 8\}$ tessellation)



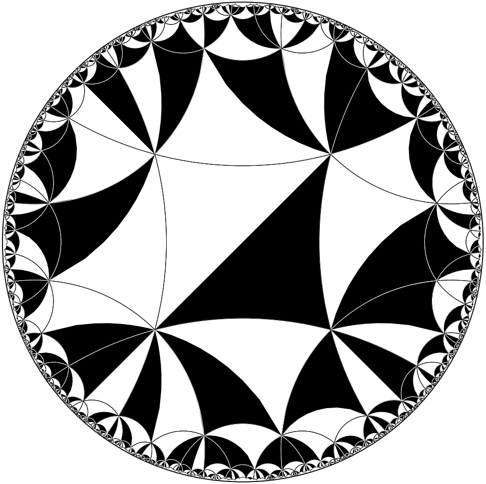
A Different Regular Hyperbolic Truchet Tiling
(based on the $\{4,6\}$ tessellation)



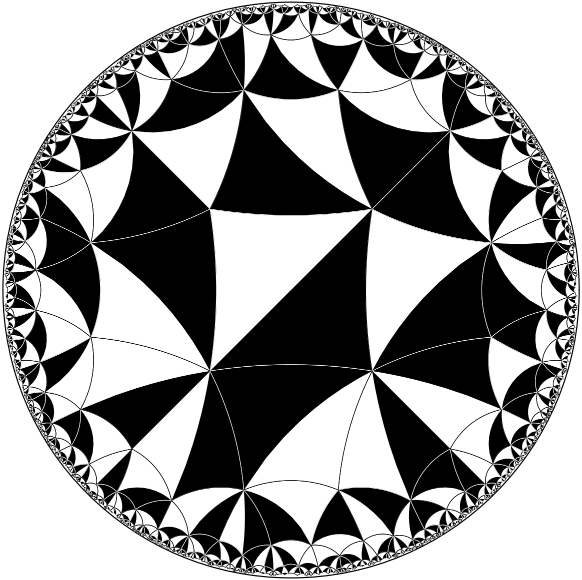
A Non-Regular Hyperbolic Truchet Tiling
(based on the $\{4, 5\}$ tessellation)



Random Hyperbolic Truchet Tilings
(One based on the $\{4, 6\}$ tessellation)

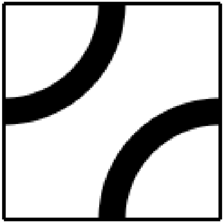


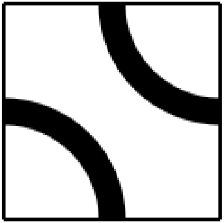
Another Random Hyperbolic Truchet Tiling
(based on the $\{4, 5\}$ tessellation)



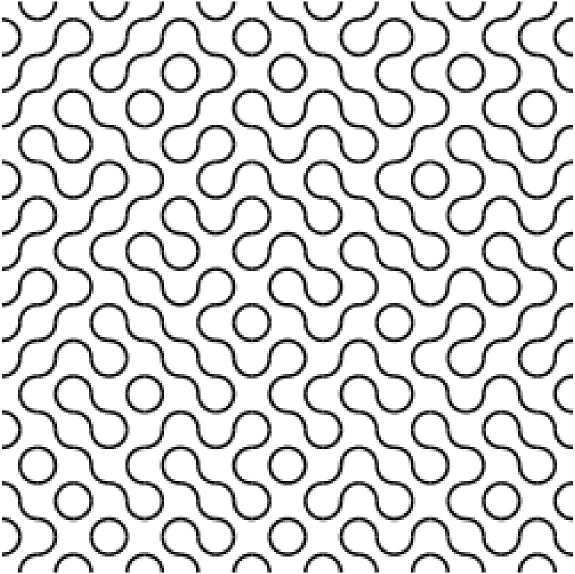
Hyperbolic Circular Arc Truchet Tilings

- Truchet circular arc tilings
- Based on a square with circular arcs connecting adjacent sides — 2 orientations.
- Either repeating patterns or random patterns.

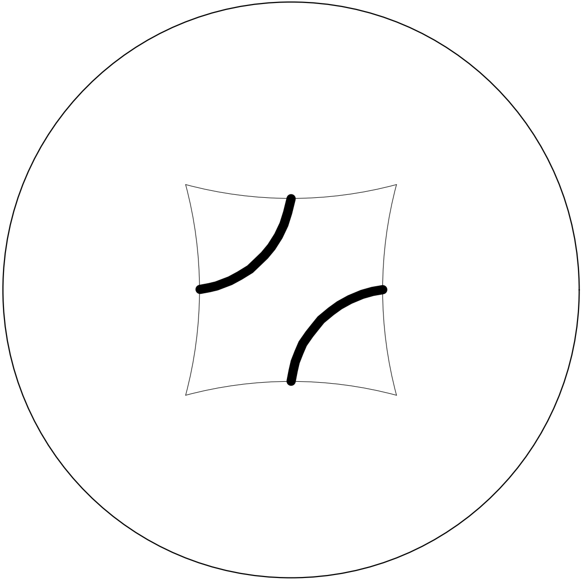




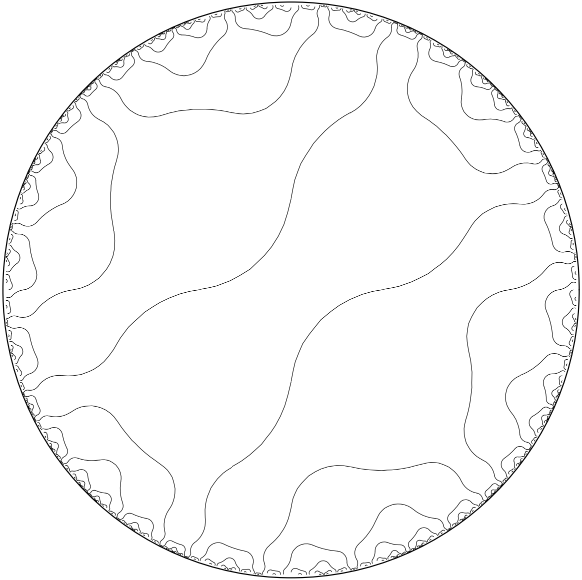
A Random Truchet Arc Tiling
(based on the $\{4, 6\}$ tessellation)



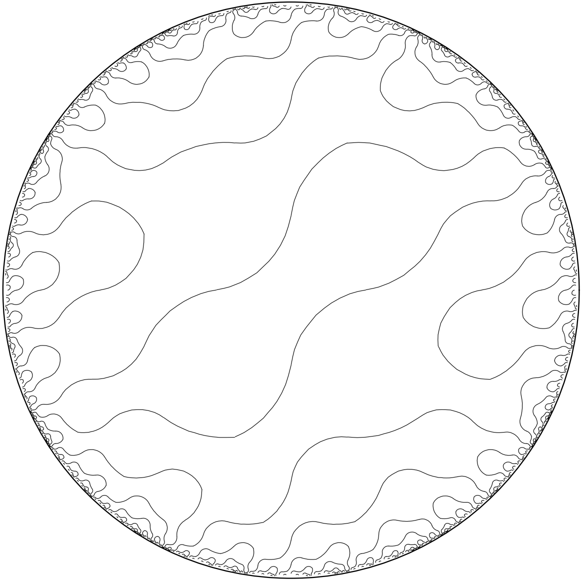
A Hyperbolic Arc Tile (based on the $\{4, 6\}$ tessellation)



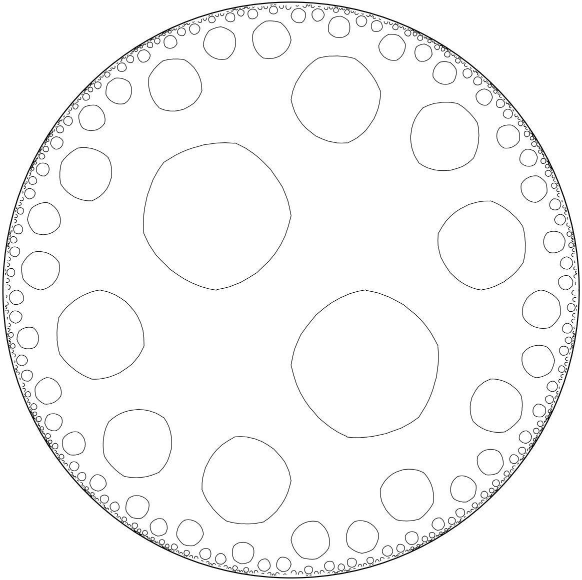
A Hyperbolic Arc Pattern (based on the $\{4, 6\}$ tessellation)



A Hyperbolic Arc Pattern (based on the $\{4, 5\}$ tessellation)



A Hyperbolic Arc Pattern of Circles (based on the $\{4, 5\}$ tessellation)



Future Work

- Try coloring hyperbolic Truchet triangle patterns.
- Implement a hyperbolic circular arc tool in the program.
- Investigate more hyperbolic Truchet arc patterns.
- Determine the number of different arc tile for a $2n$-gon.

Thank You!

Gary

And the other organizers at the AMS and University of Richmond