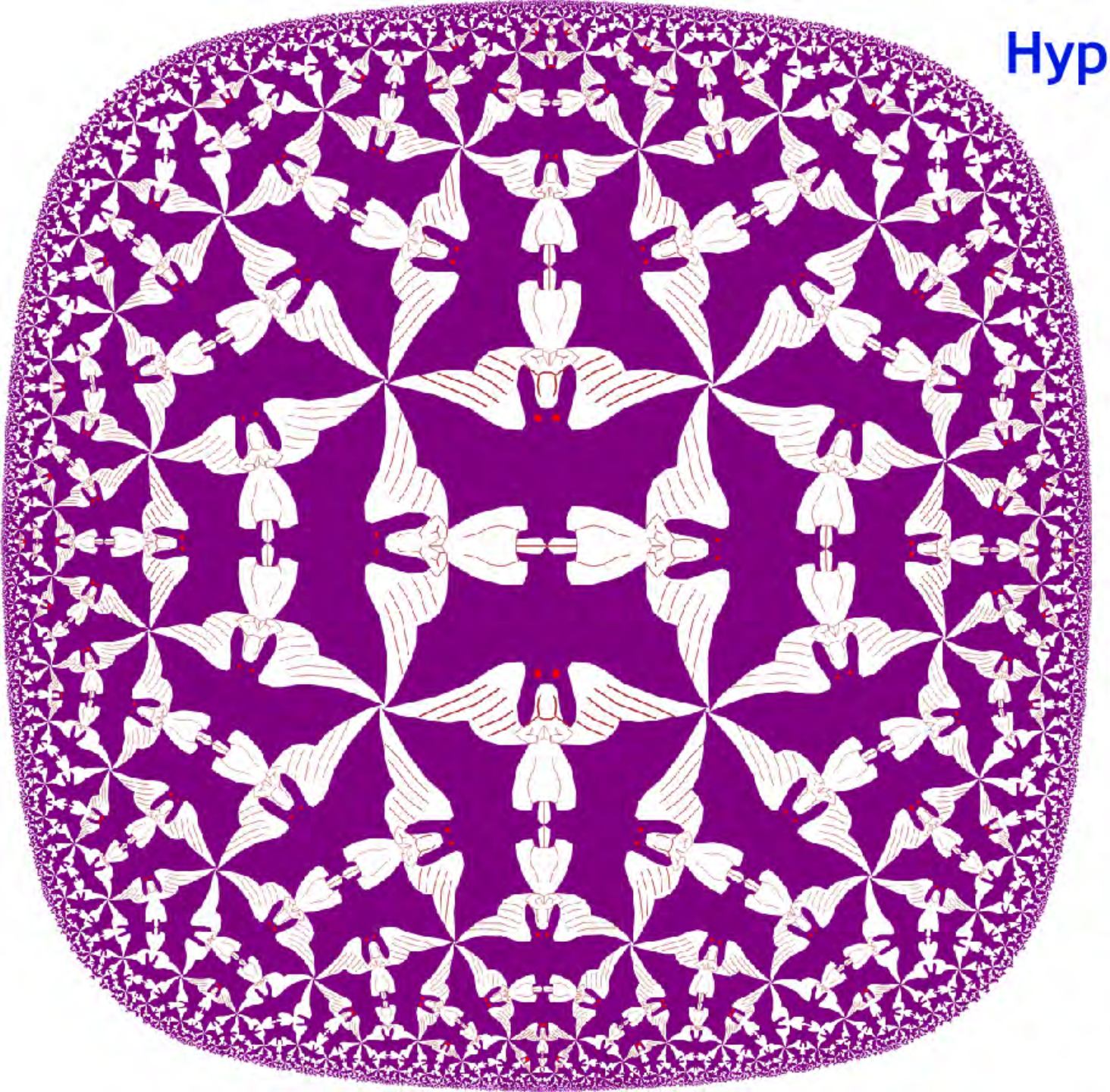


Hyperbolization on a Squircular Continuum

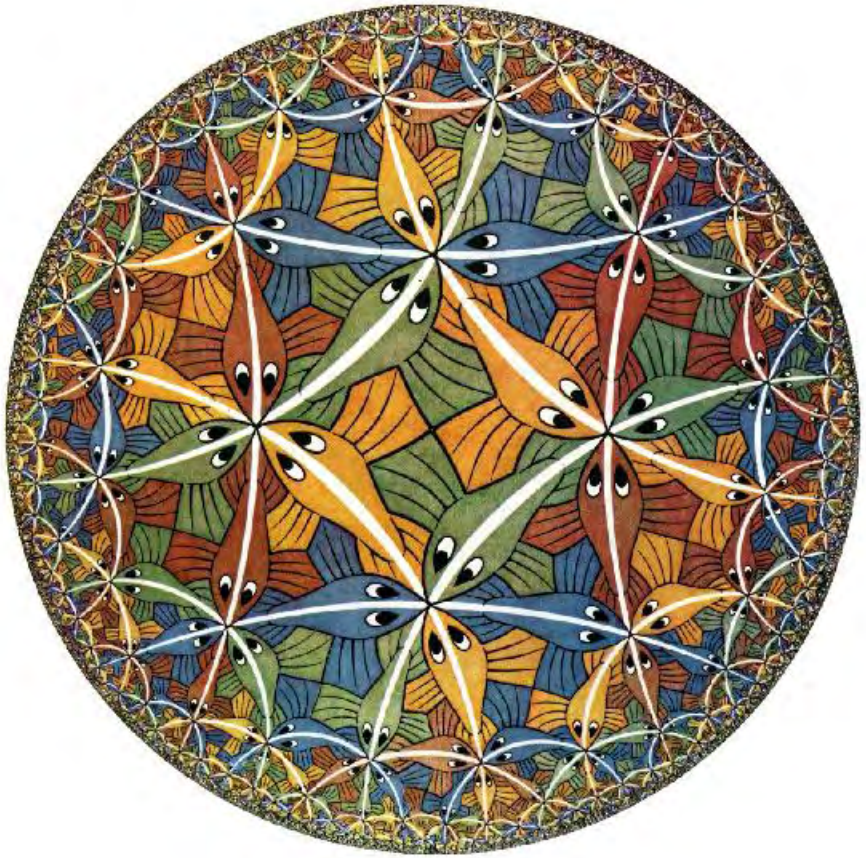
Chamberlain Fong

Douglas Dunham

JKU

BRIDGES LINZ 2019

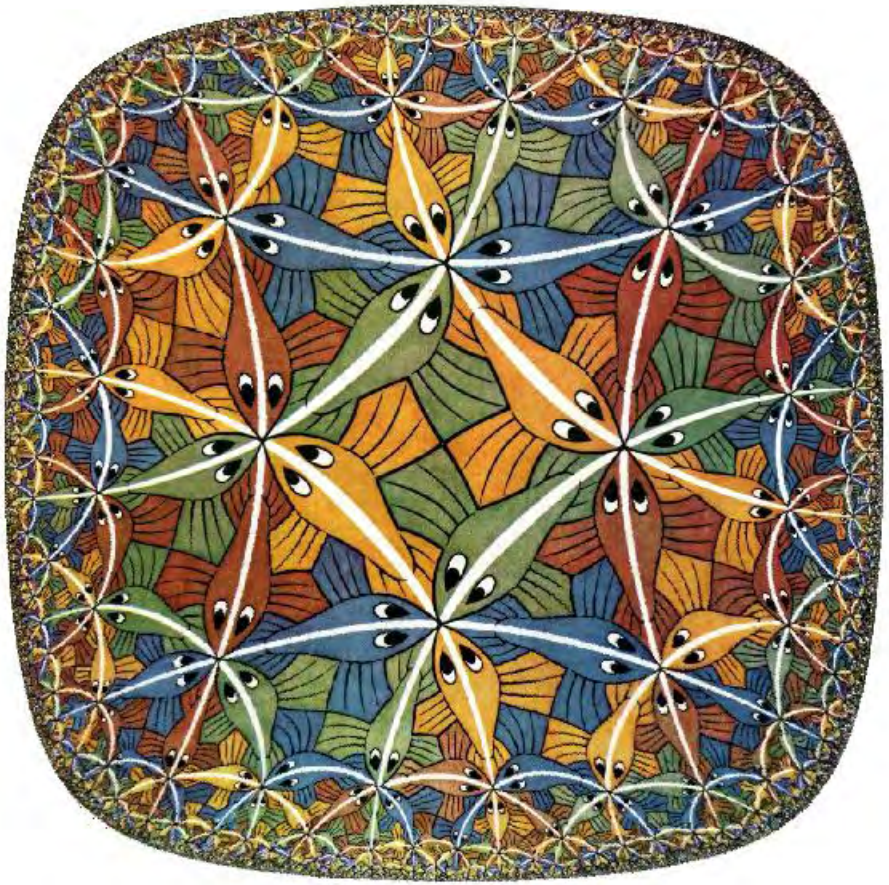
in 1959



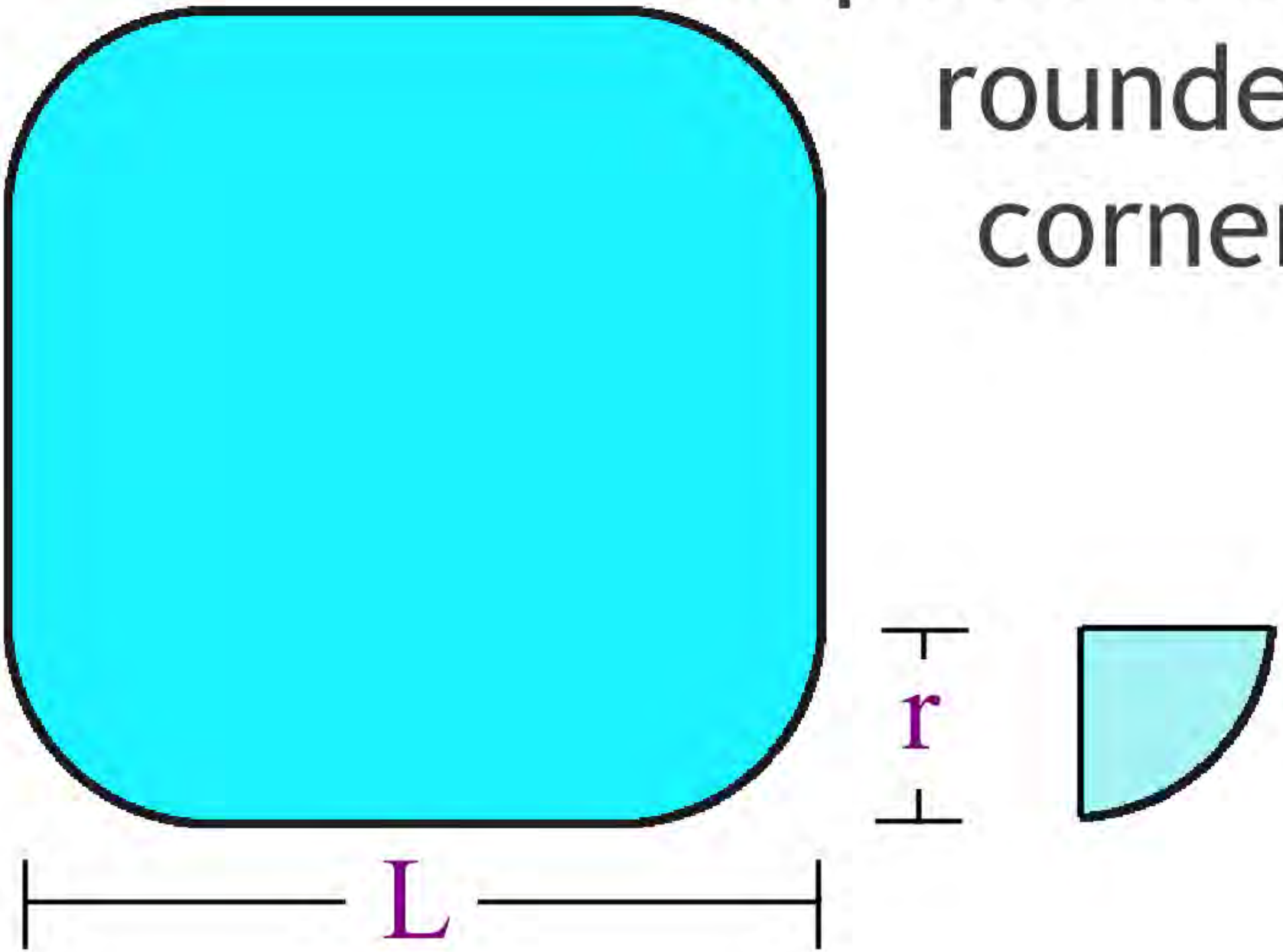
Fishes & Scales (1959)

Squircle Limit (2019)

Squircular Escher

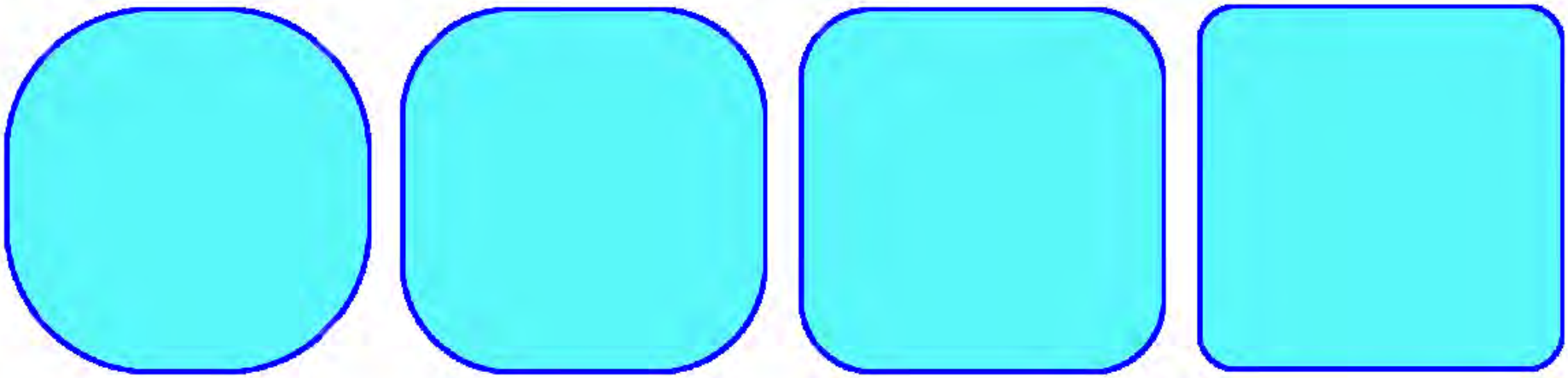


square with rounded corners



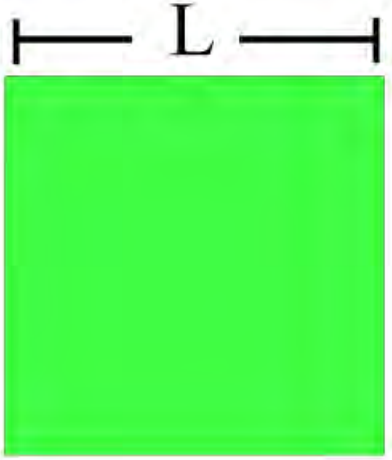
square

with rounded corners



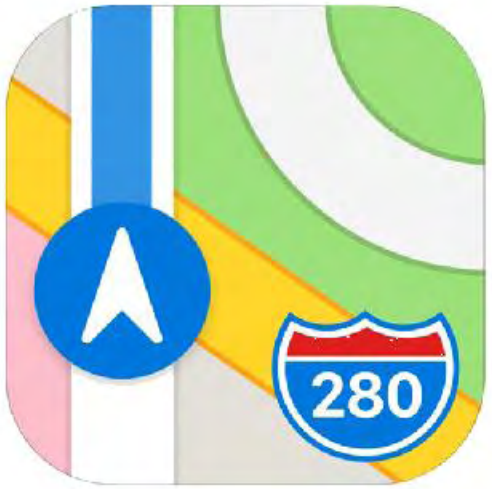
circle

square



only C^1 continuous

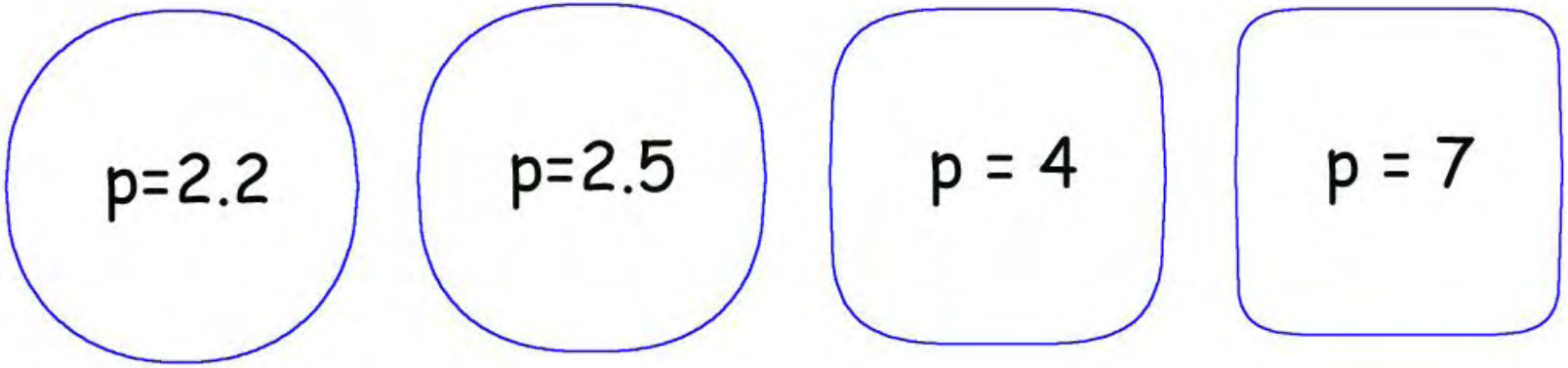
squircles everywhere



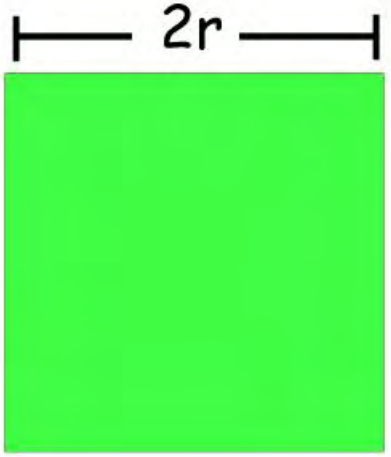
Lamé squircle

$$|x|^p + |y|^p = r^p$$

1818



circle when p = 2

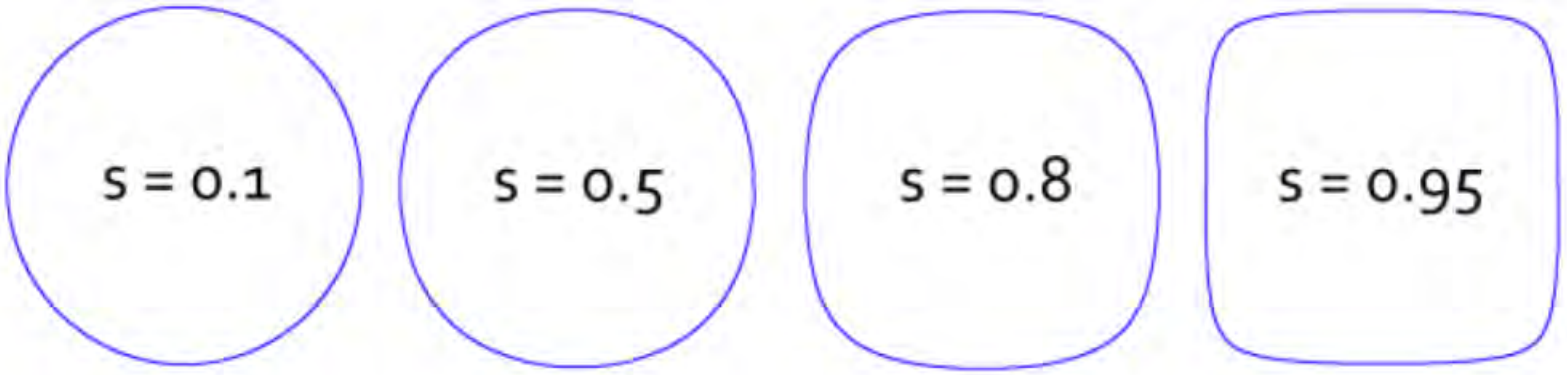


square when p → ∞

Fernandez-Guasti squircle

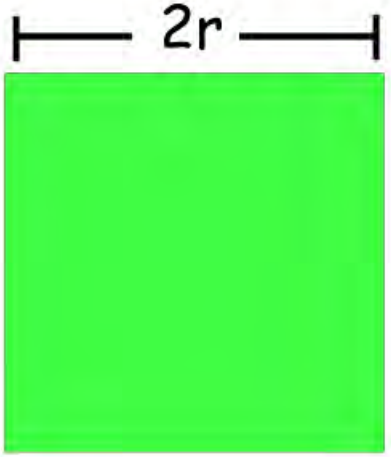
$$x^2 + y^2 - \frac{s^2}{r^2} x^2 y^2 = r^2$$

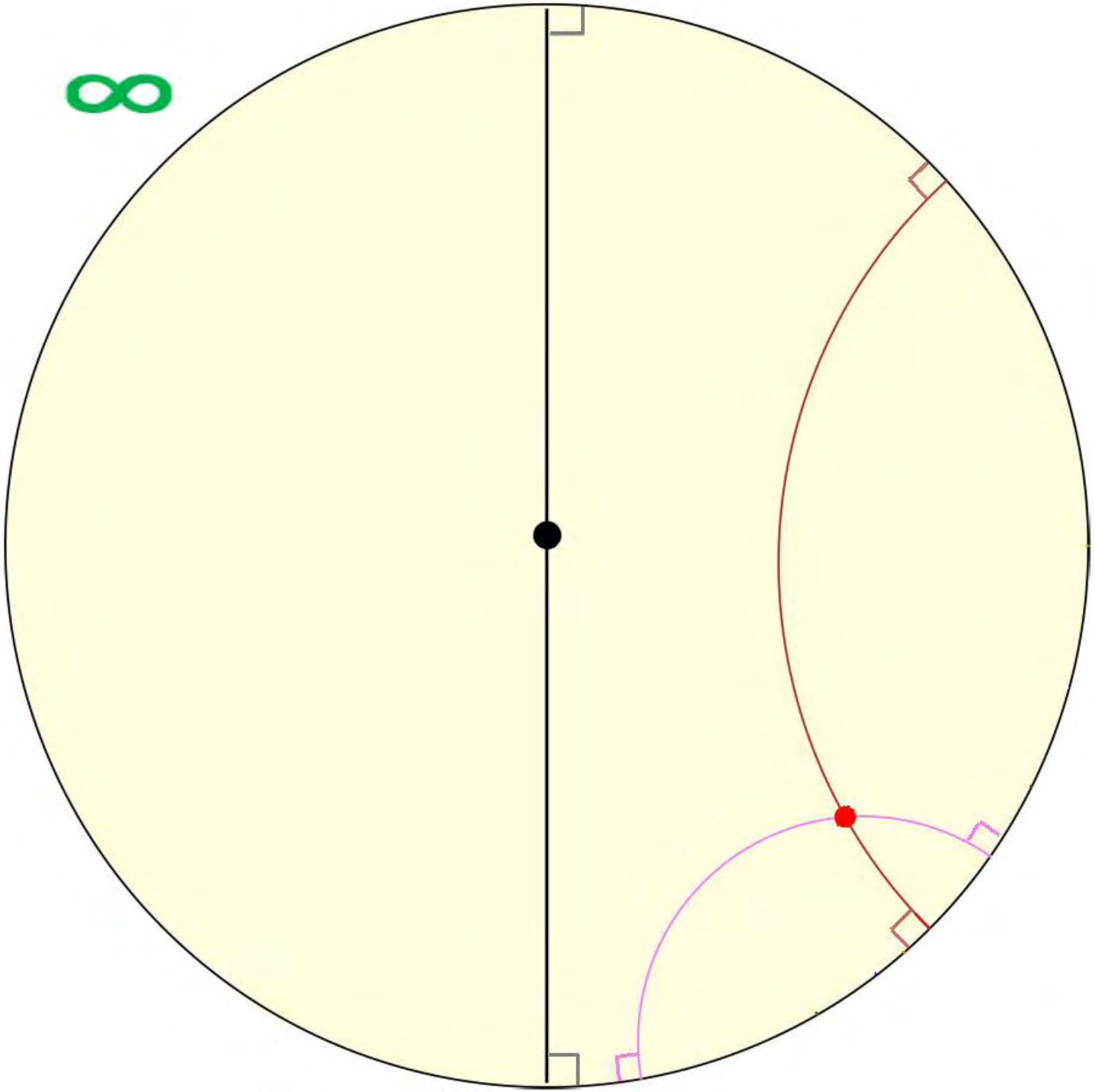
$0 \leq s \leq 1$



circle when s=0

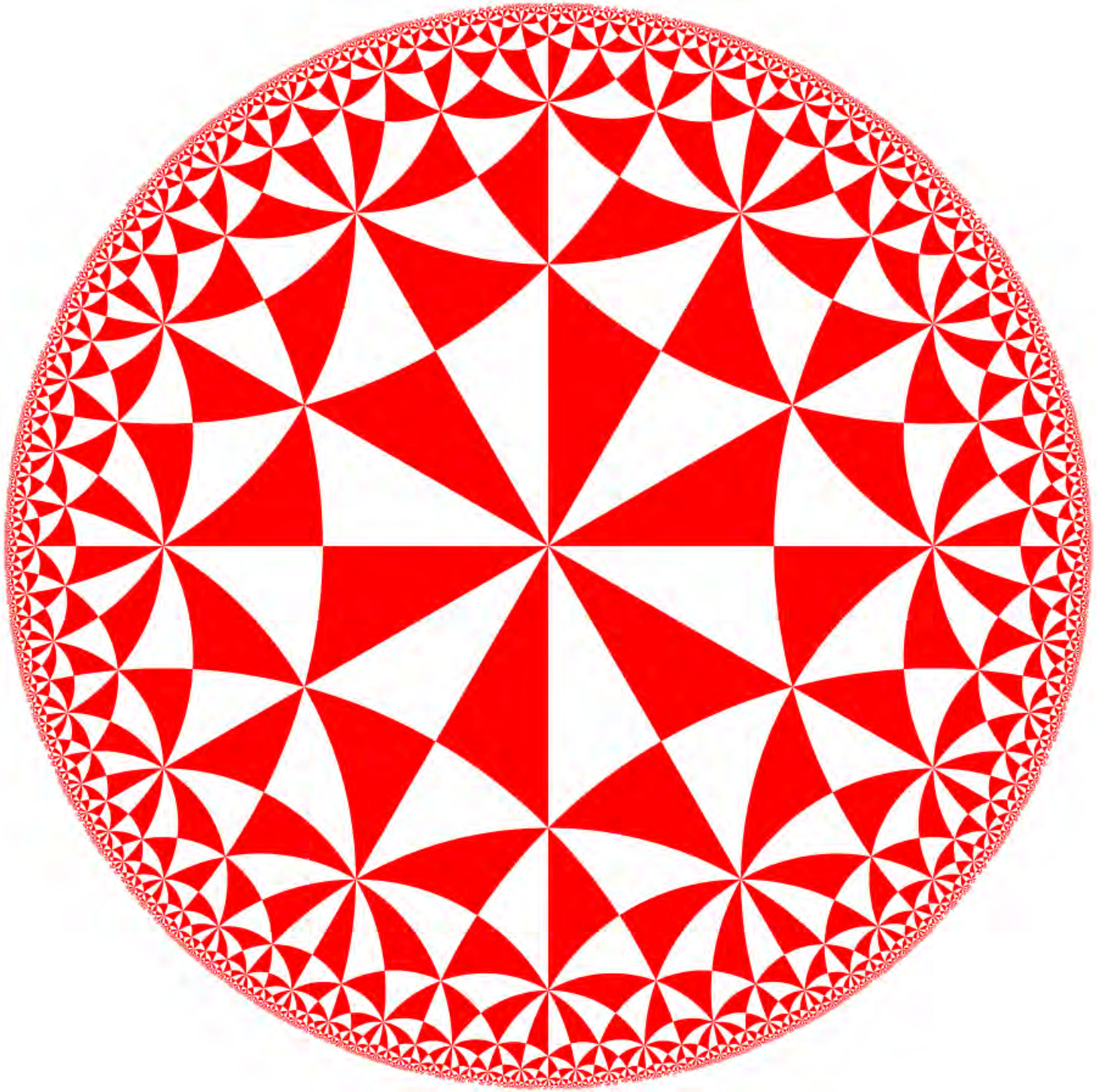
square when s=1





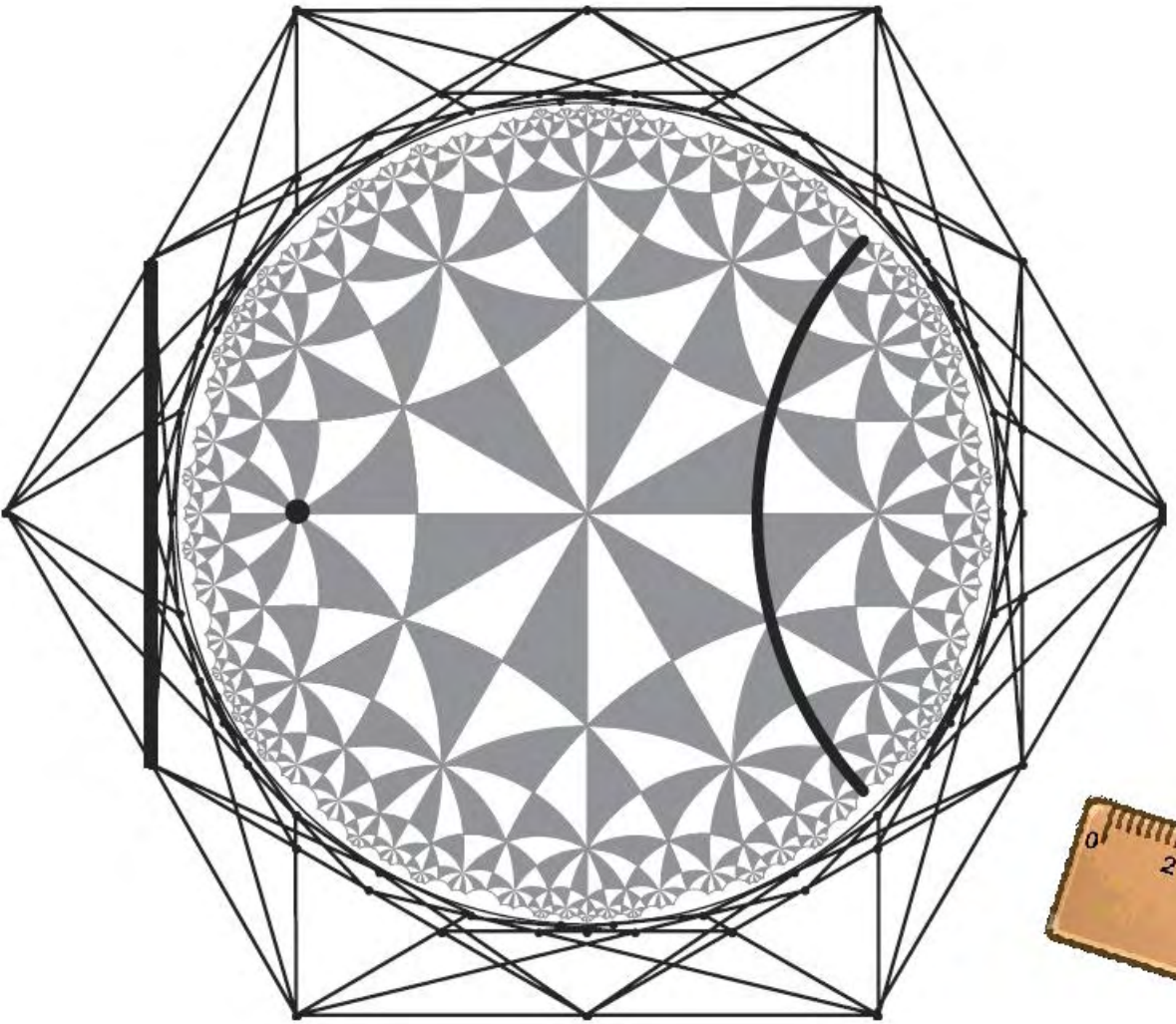
Henri
Poincare
1882

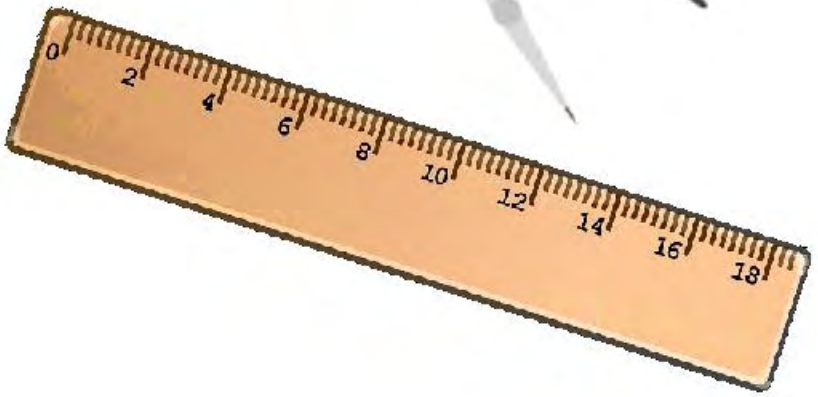
Poincare
disk
model
(conformal)



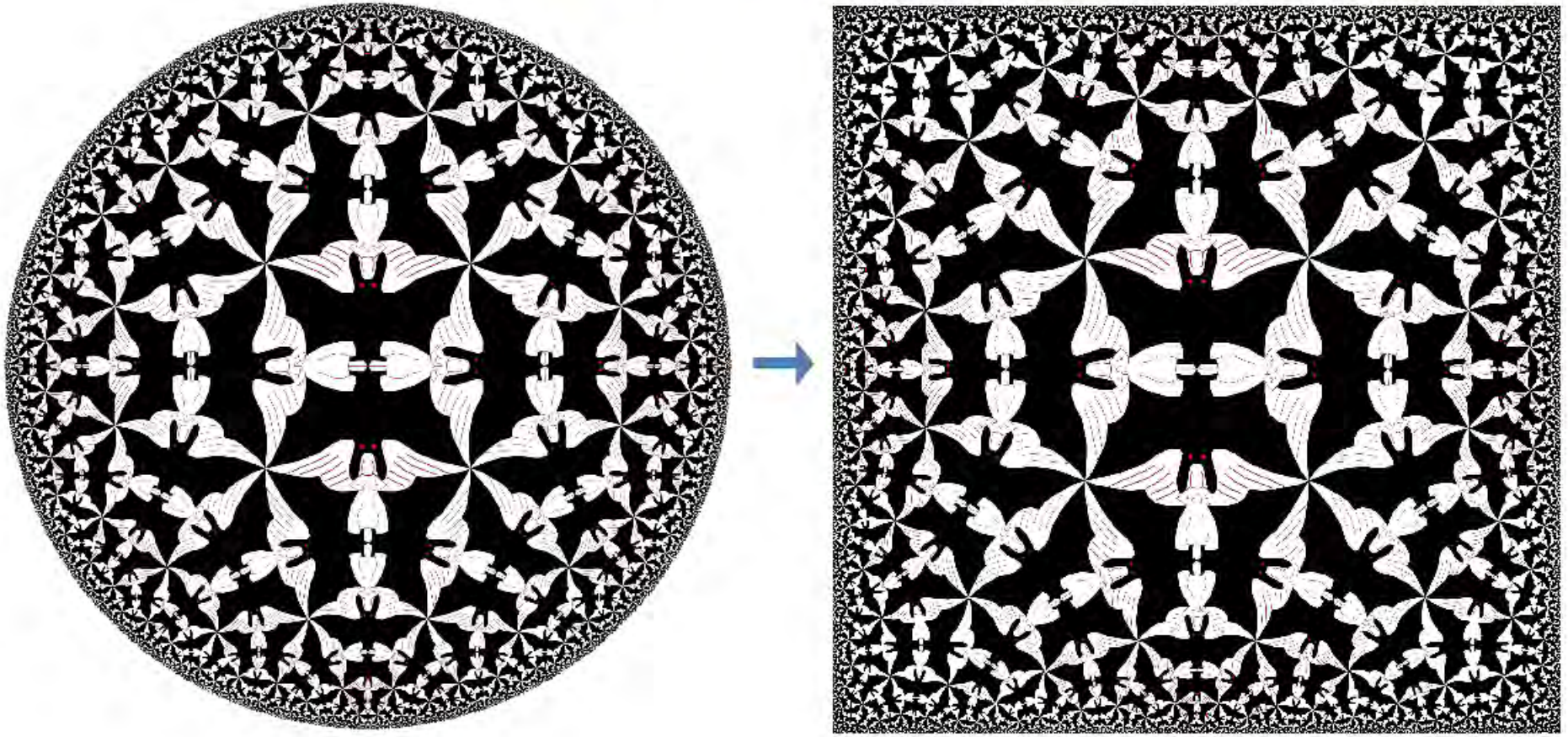
H.S.M.
Coxeter
1957

basis for
Escher's
Circle
Limits




0
2
4
6
8
10
12
14
16
18

conformal square

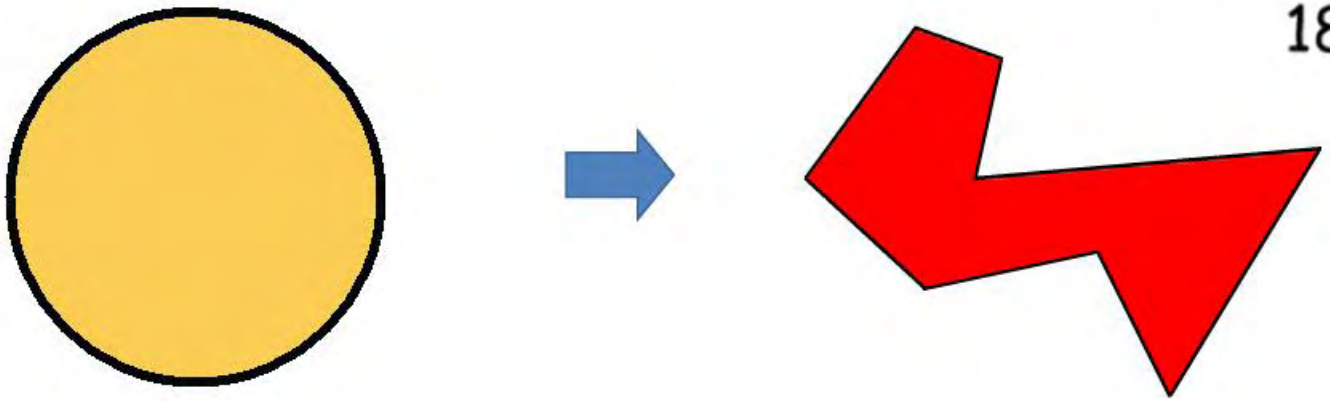


via Schwarz-Christoffel

SCHWARZ-CHRISTOFFEL MAPPING

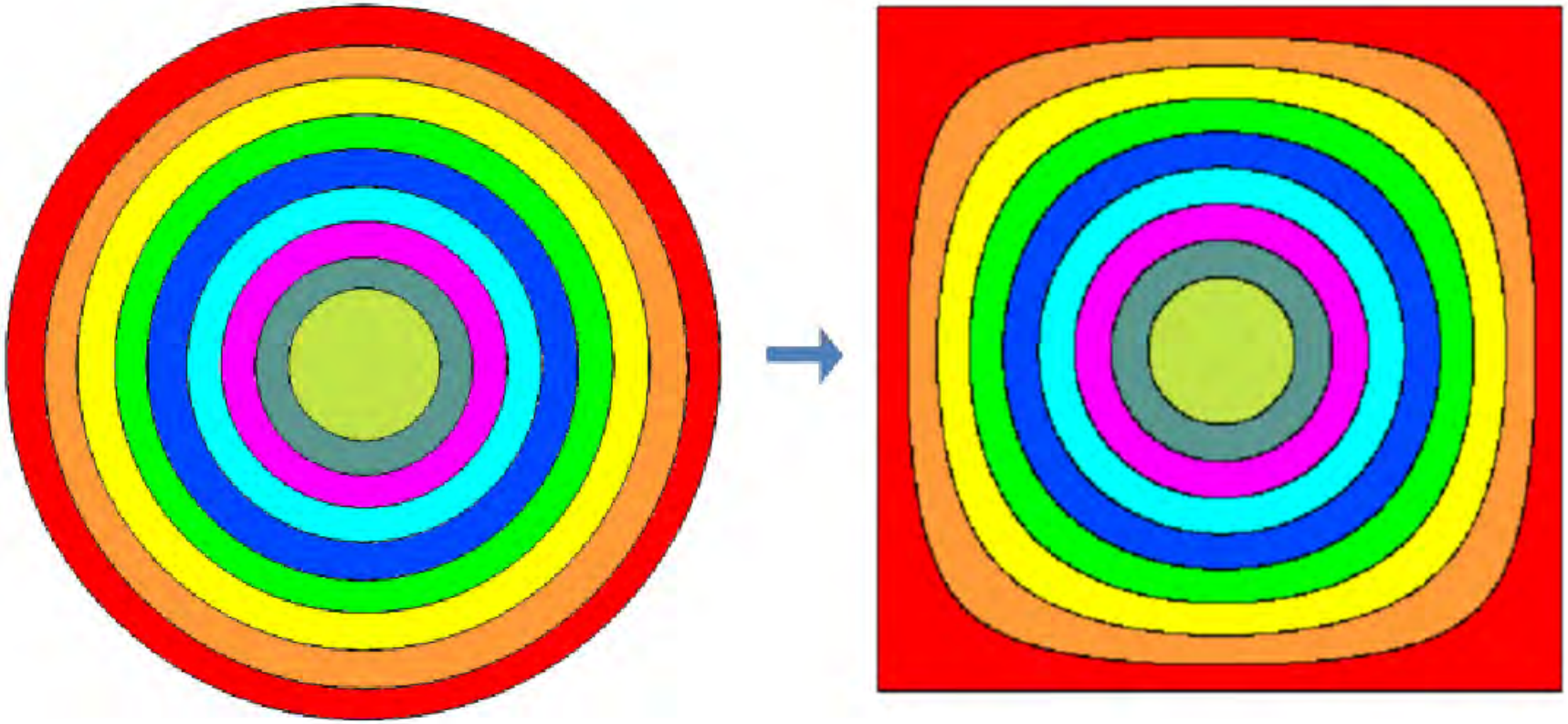
Elwin Christoffel
1867

Hermann Schwarz
1869



$$f(z) = A + C \int^{z} \prod_{k=1}^{n} \left(1 - \frac{\zeta}{z_k}\right)^{\alpha_k - 1} d\zeta$$

squircular continuum



Mapping a circle to a squircle

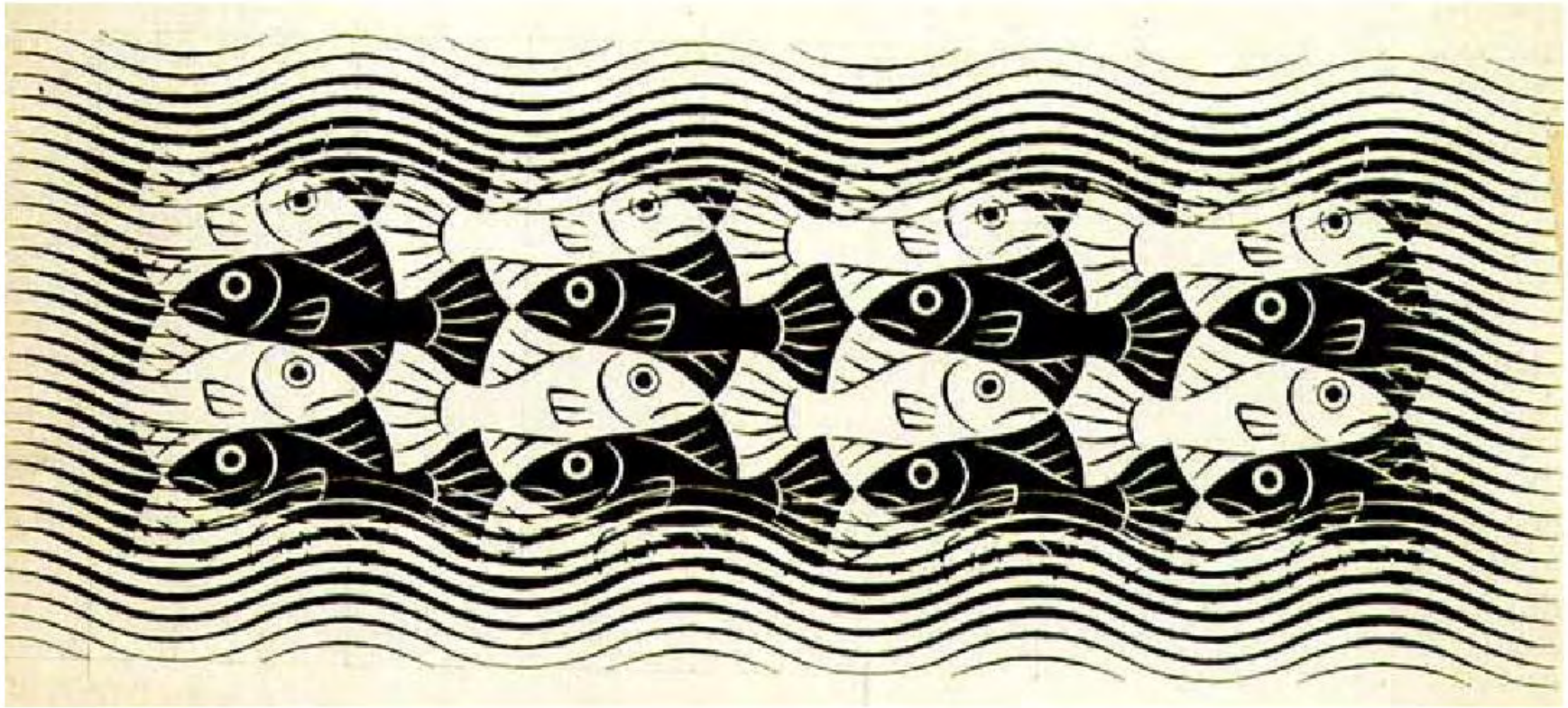
input shrunken mapped

Escher, the geometer

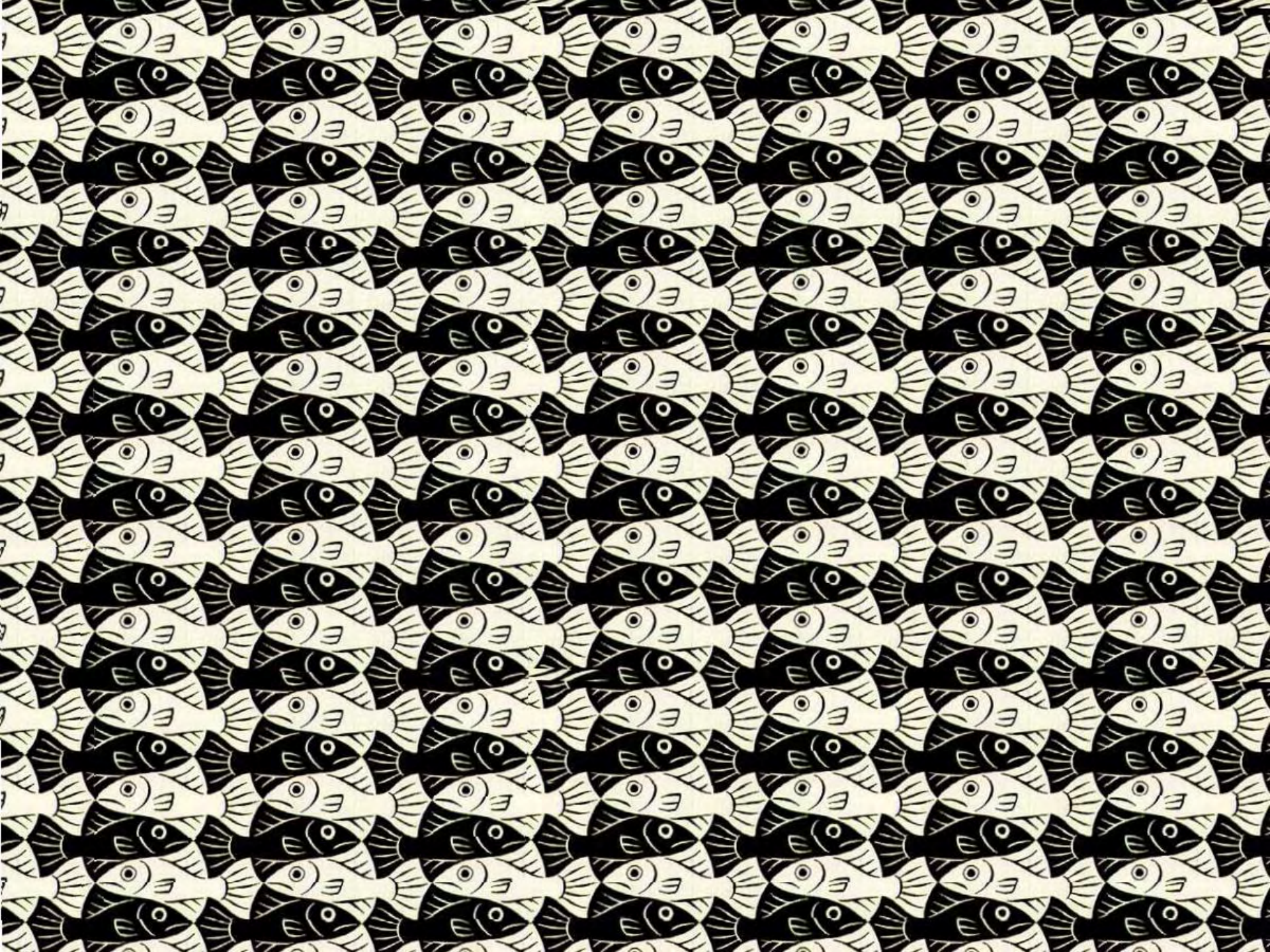
pre-1975
fractal

Escher A13

(Baarn 1958)

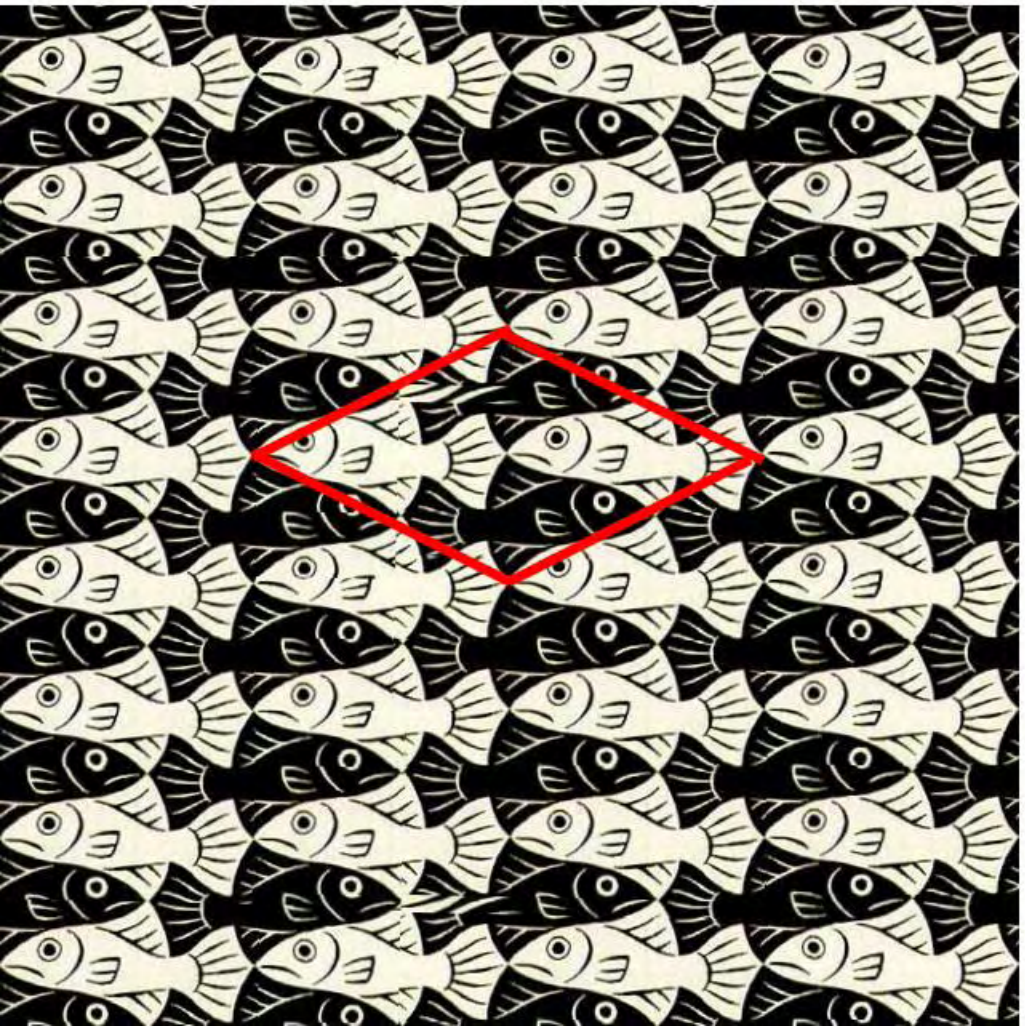


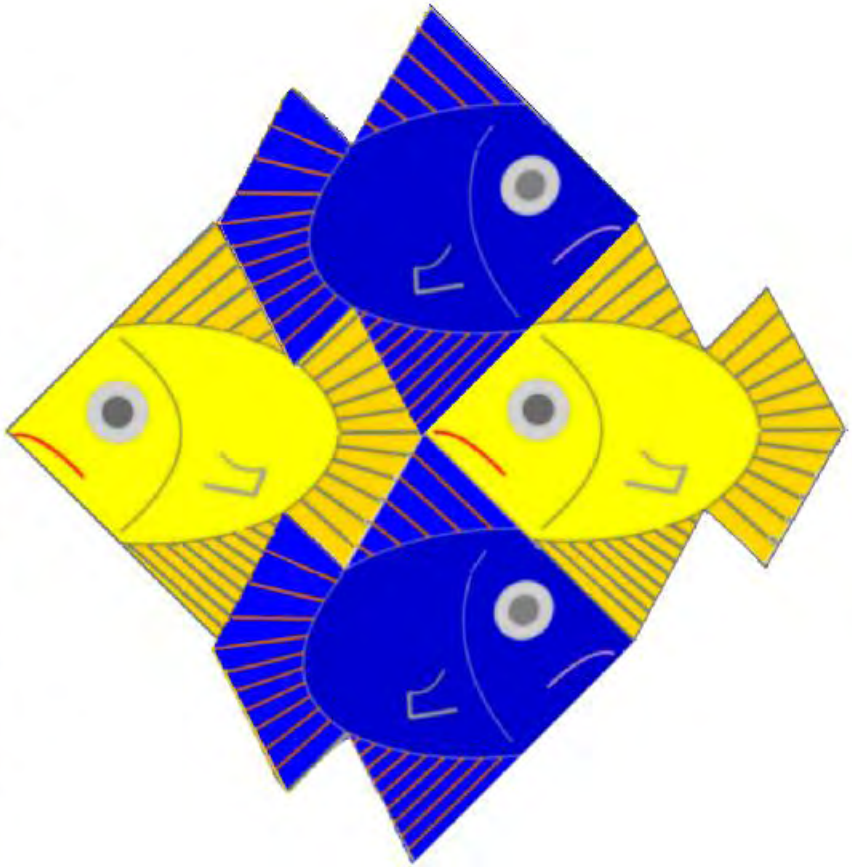
source: D. Schattschneider "Visions of Symmetry"



basic motif

PG wallpaper

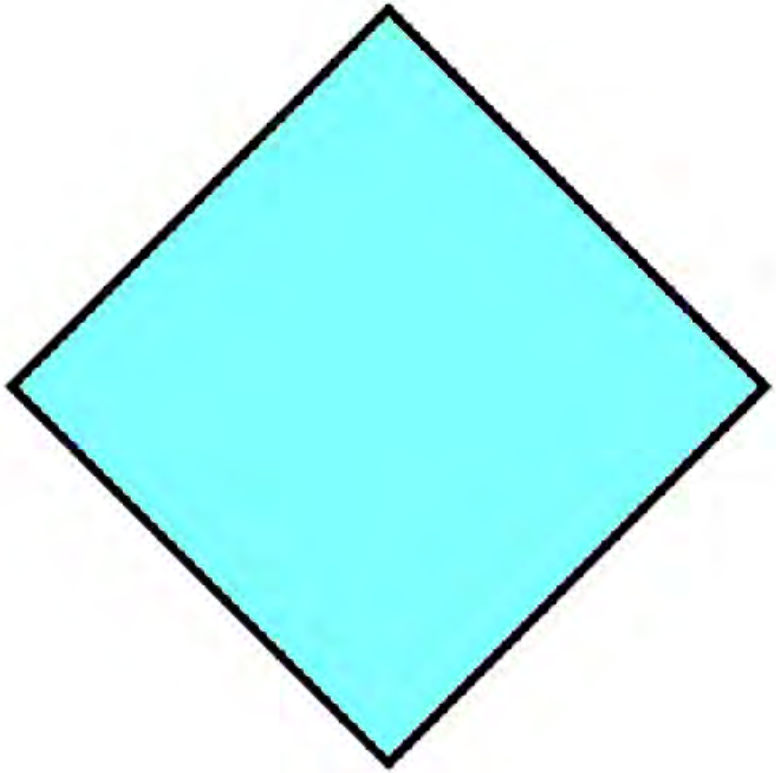


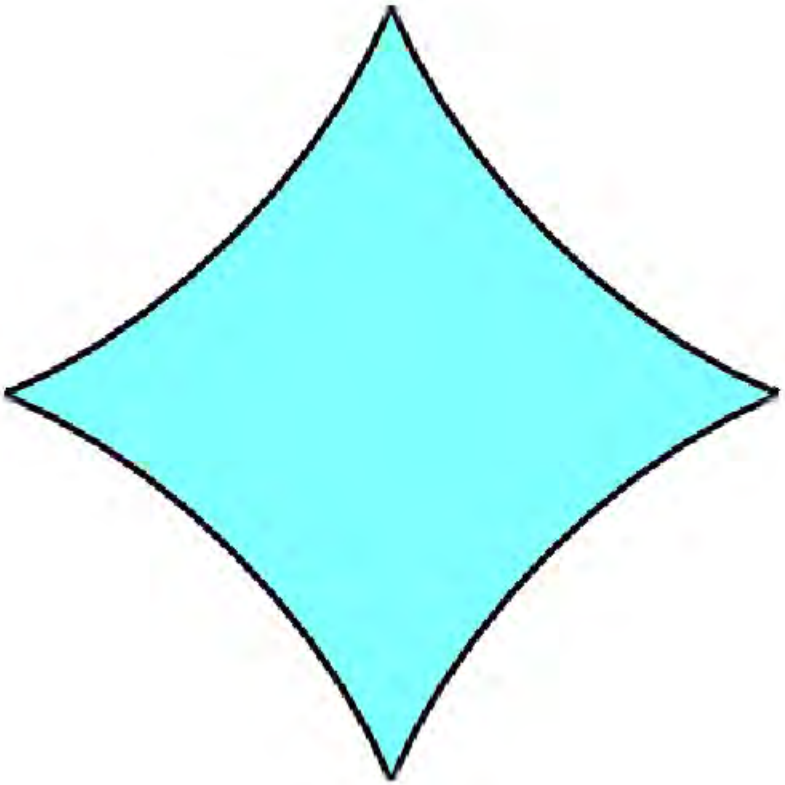


4 fishes

Euclidean

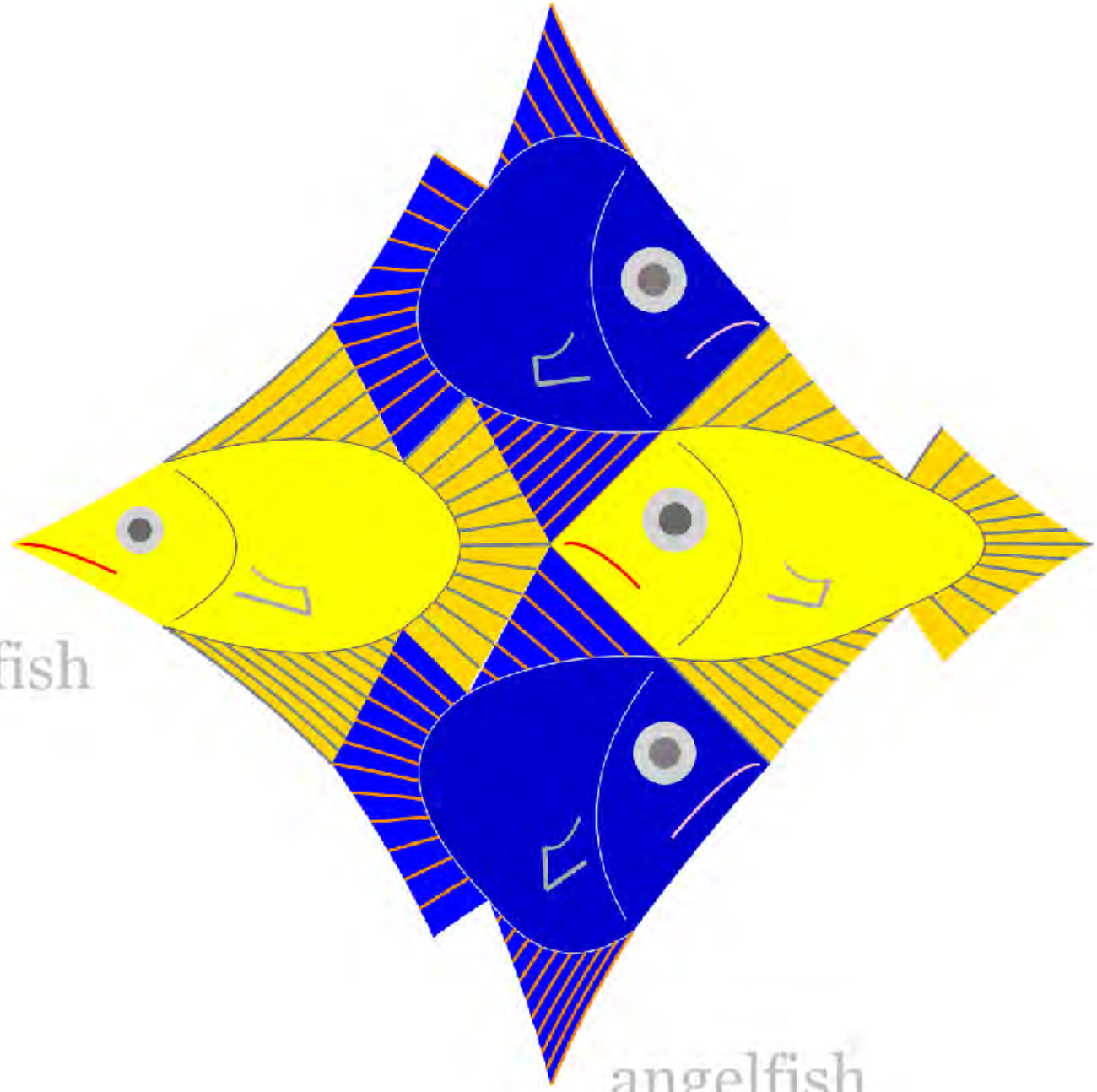
hyperbolic

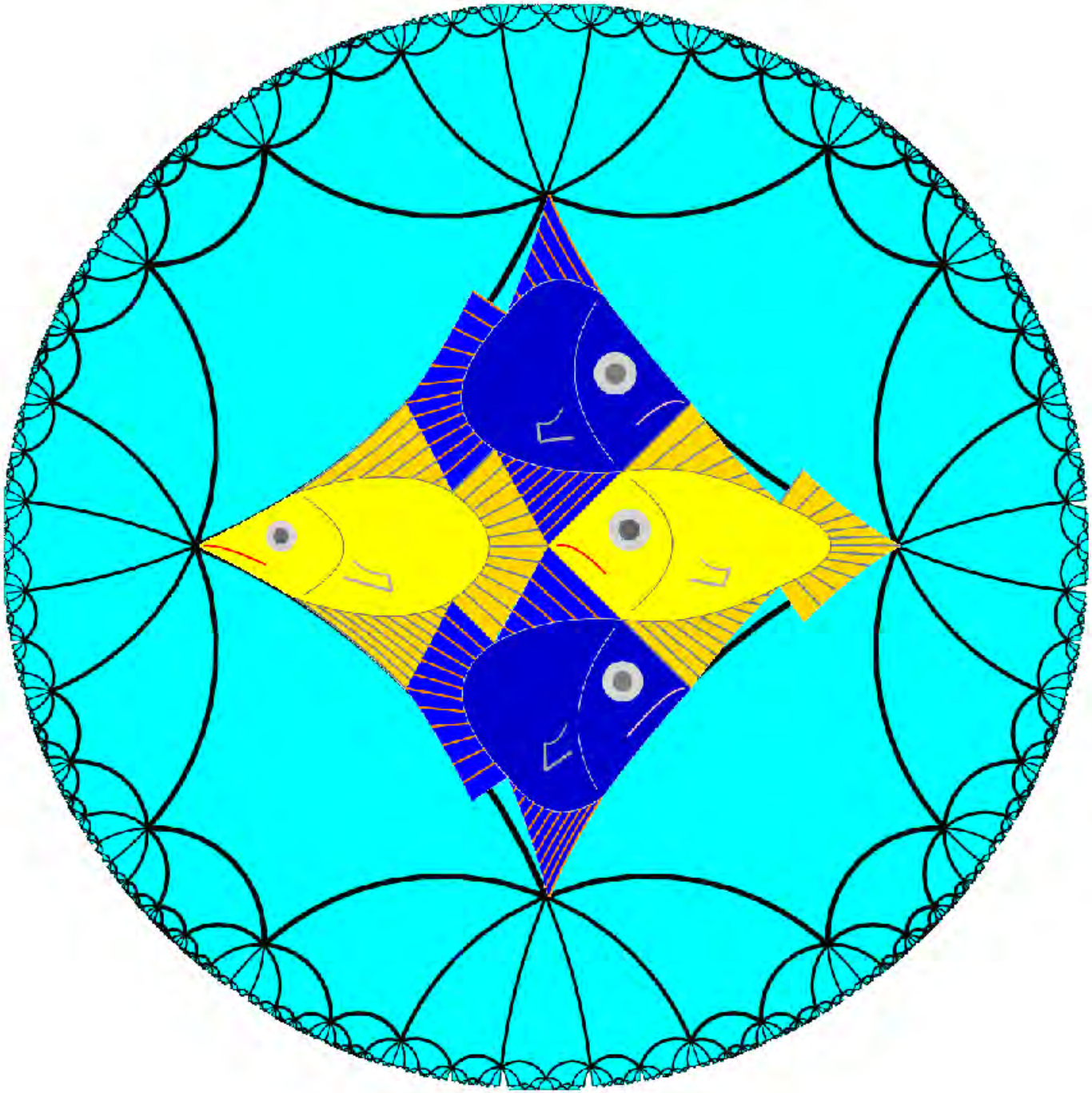


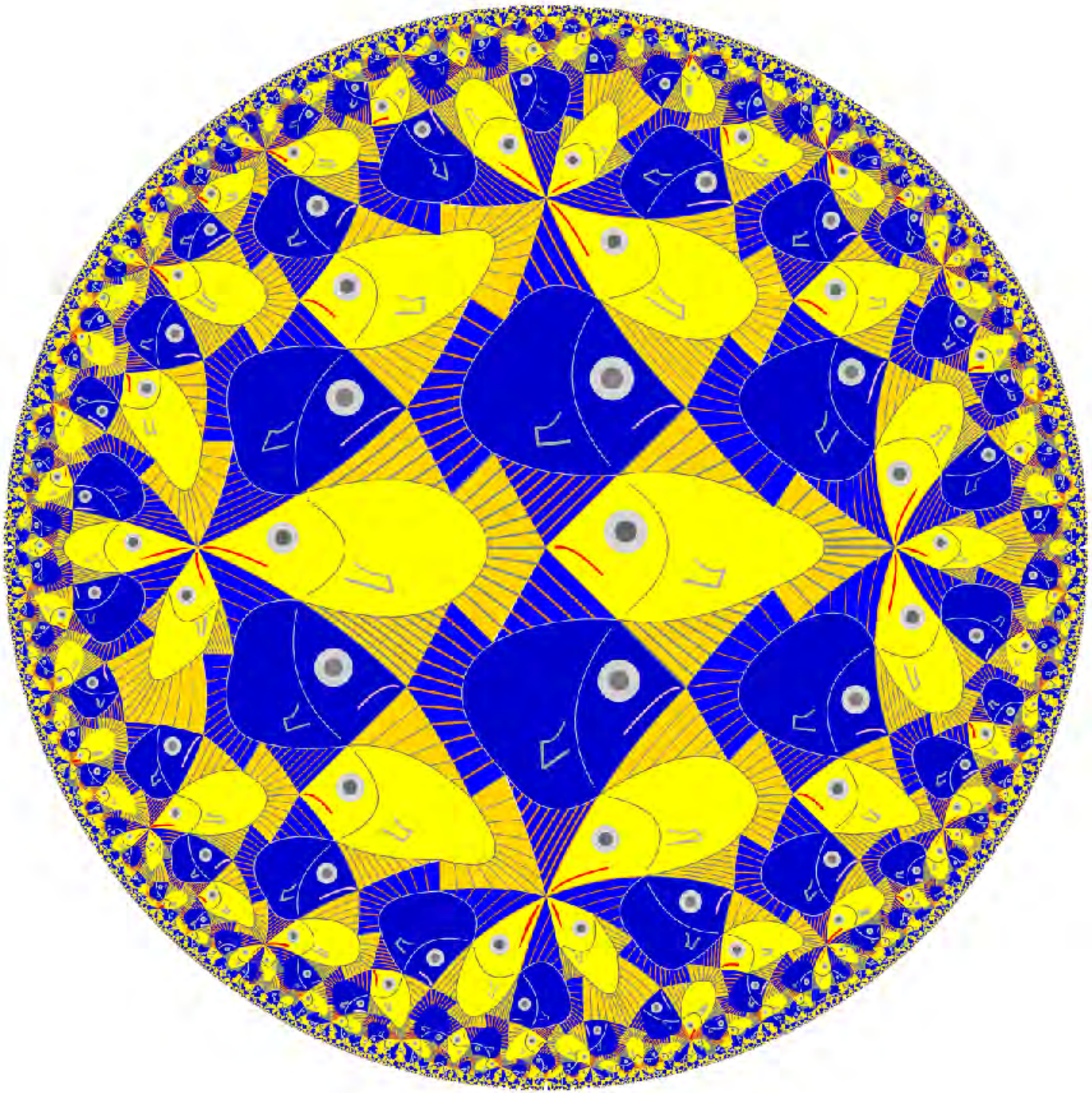


90°

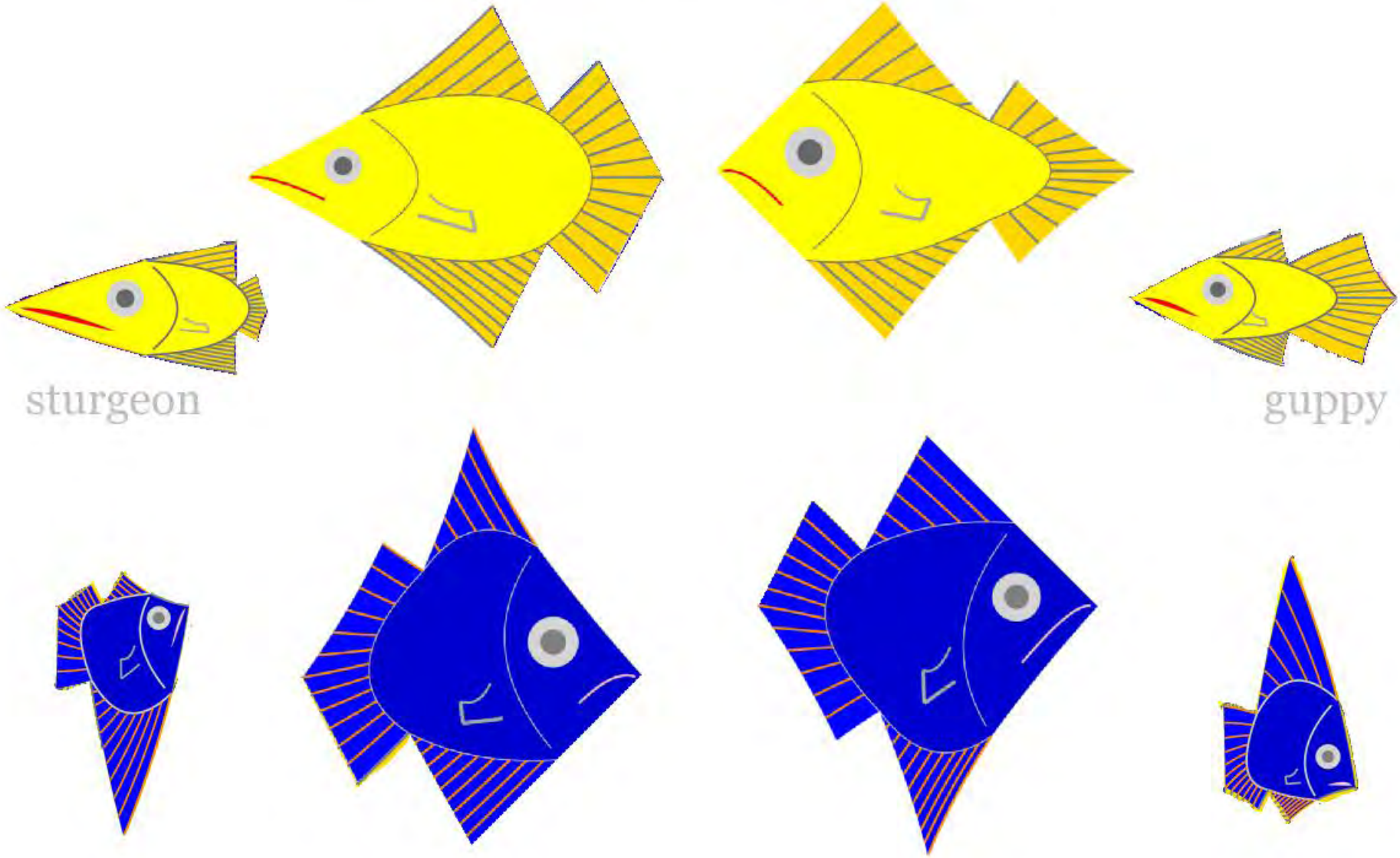
cusps


swordfish
angelfish

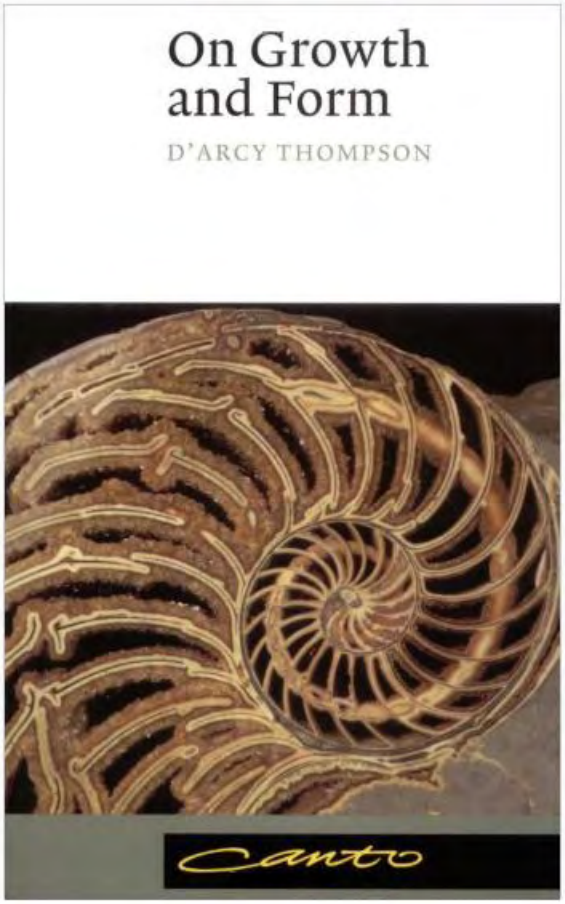




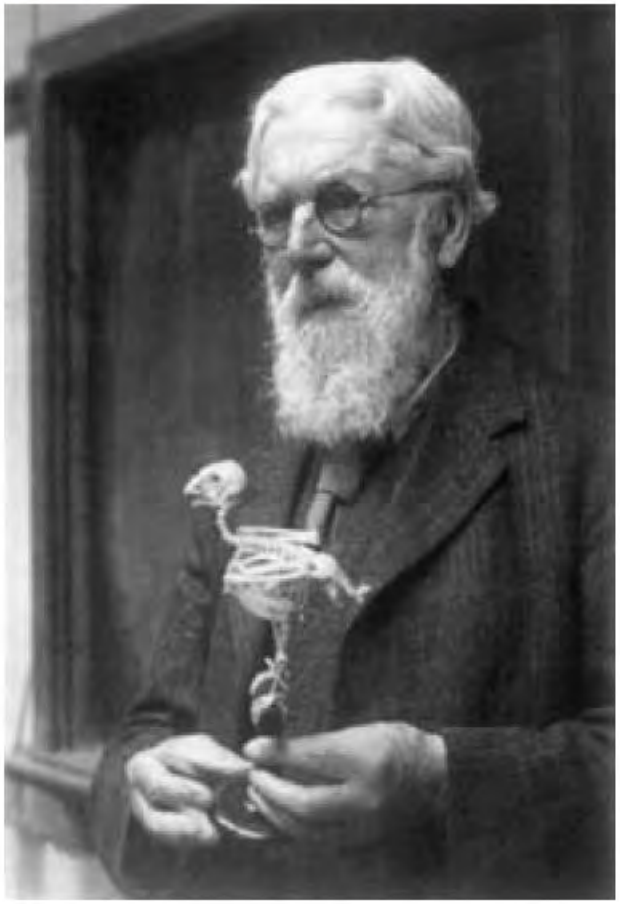
fishy variations



D'Arcy Thompson



1917



revised 1942

evolution of fish species

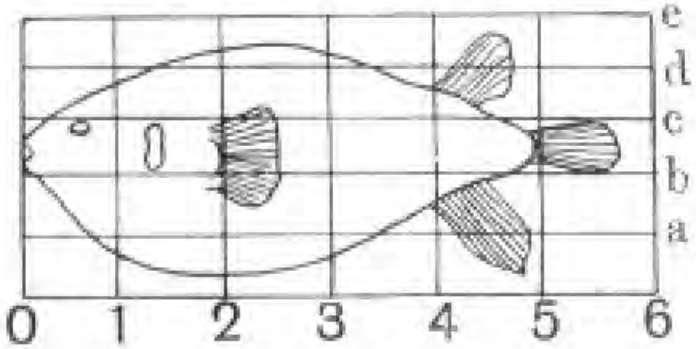


Fig. 381. *Diodon.*

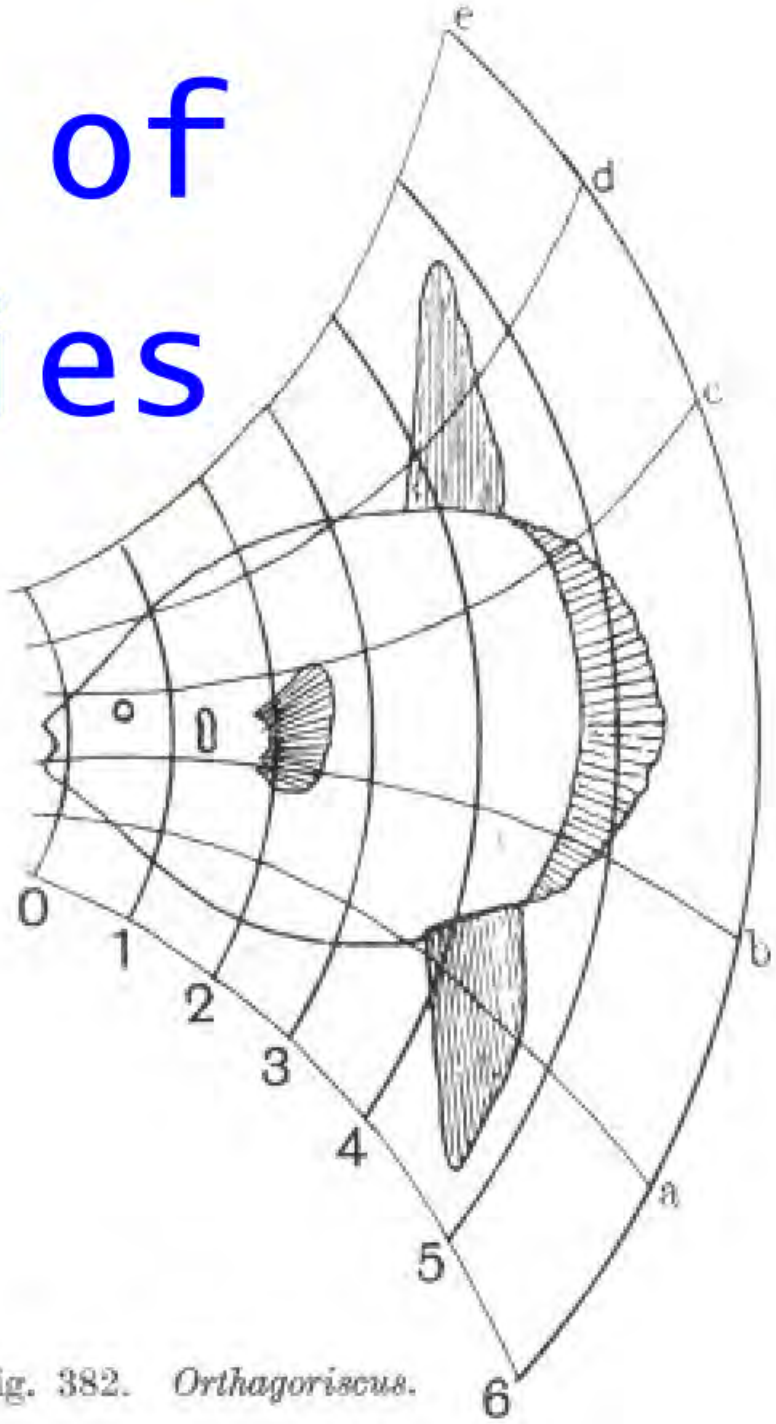


Fig. 382. *Orthagoriscus.*

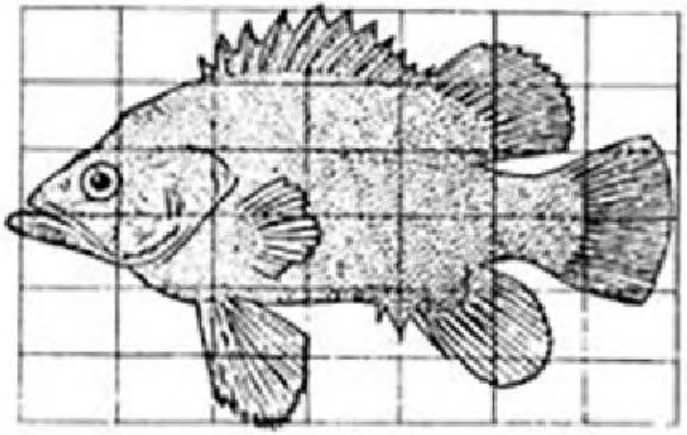


Fig. 150. *Polyprion.*

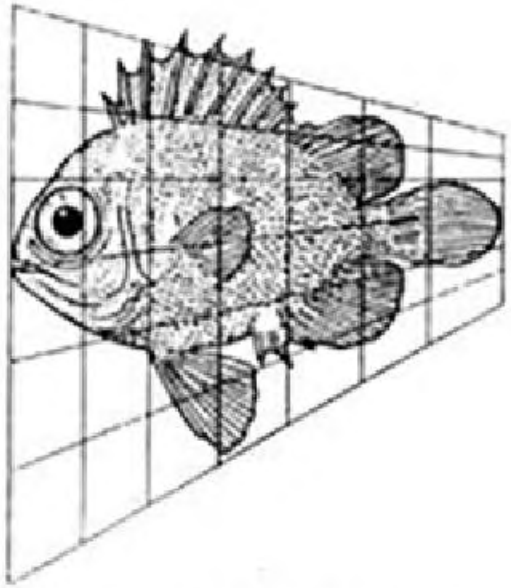


Fig. 151. *Pseudopriacanthus altus.*

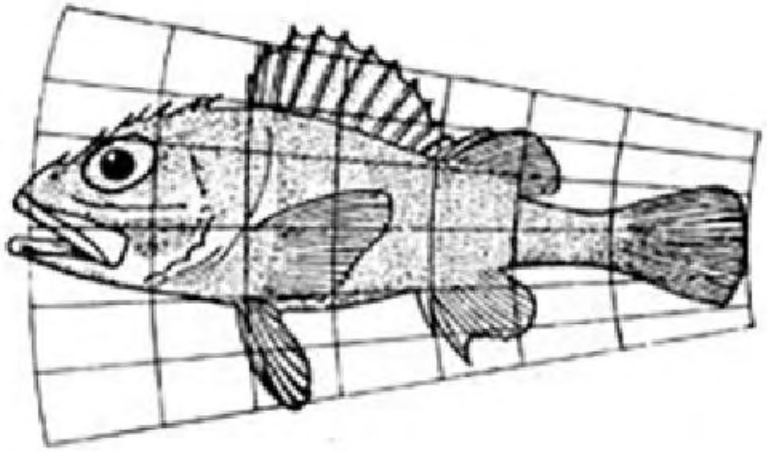


Fig. 152. *Scorpaena* sp.

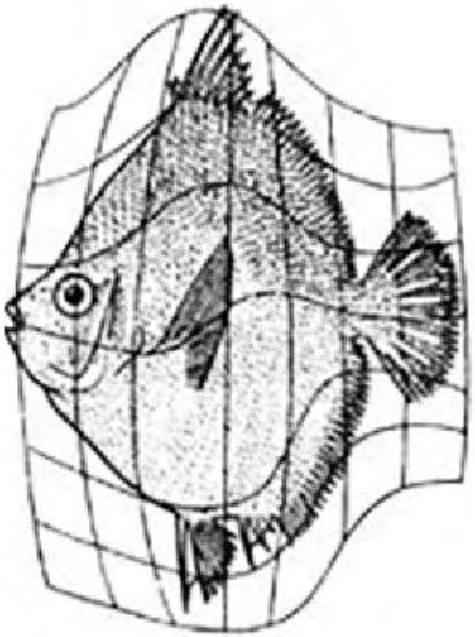
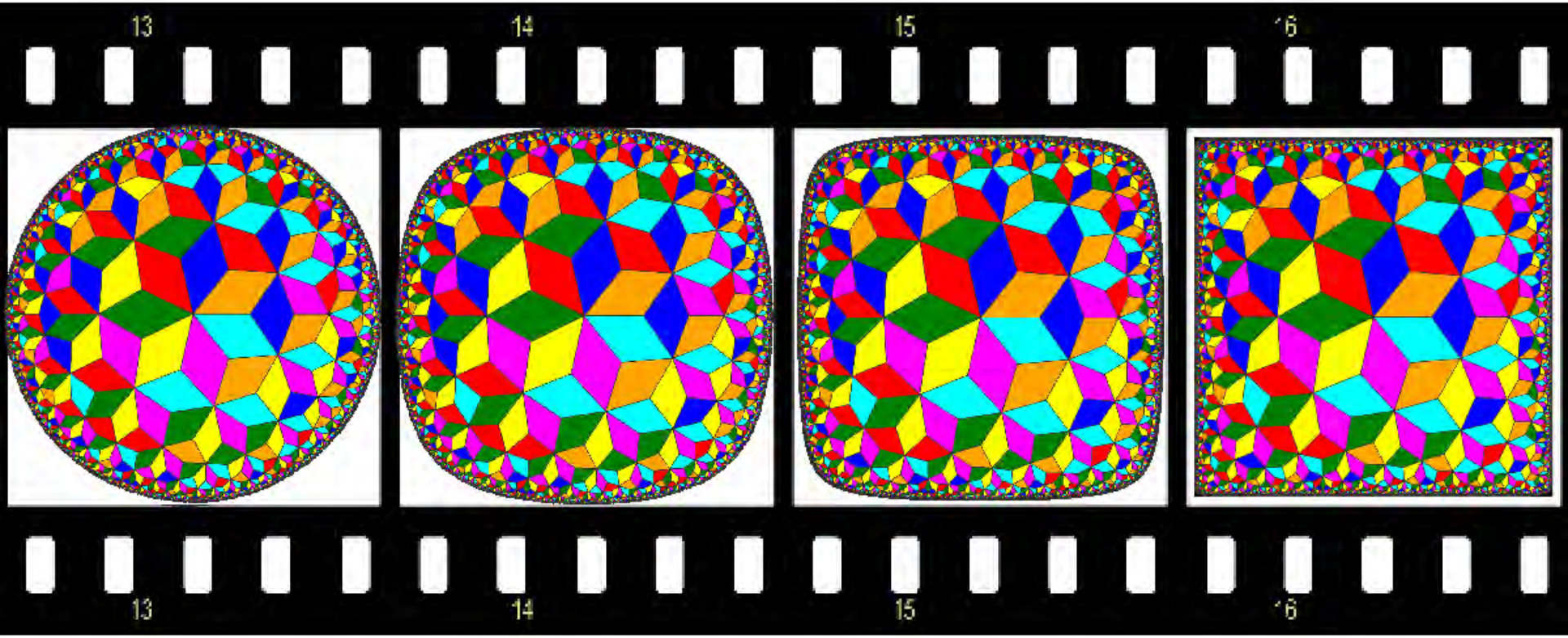
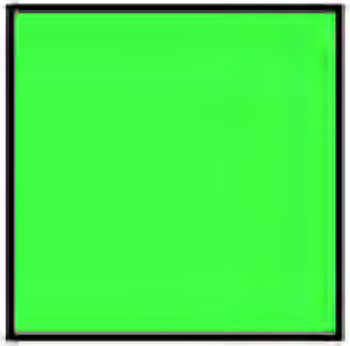


Fig. 153. *Antigonia capros.*

morphing animations



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questions

?

thanks to

Susan Goldstine

& the anonymous reviewers