Symmetry Festival, Sofia, Bulgaria, July 2021

Creating Symmetric Art Using Craft Technologies

Douglas Dunham
Dept. of Computer Science
Univ. of Minnesota, Duluth
Duluth, MN 55812, USA

Lisa Shier
University of Maryland
Global Campus
New Market, AL 35761, USA

Outline

- Craft technologies
- An embroidered hyperbolic fish pattern
- A papercrafted hyperbolic pattern of shells
- A papercrafted part of an infinite polyhedron
- An embroidered and papercrafted dodecahedron
- Conclusions and future work
- Contact information

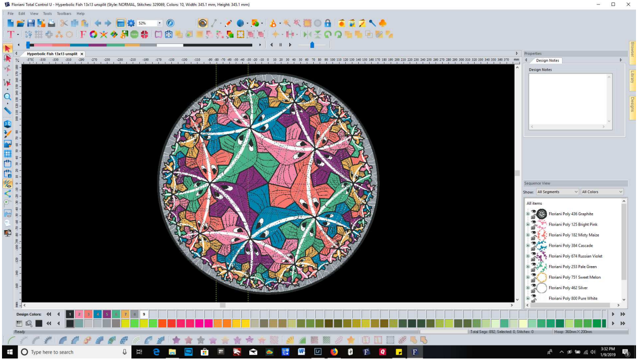
Craft Technologies

We used two computer-controlled machines and associated software to create input files for them:

- A Viking Husqvarna Epic embroidery machine using Floriani Total Control embroidery software to create the input file.
- A Brother ScanNCut SDX225 cutter/scorer/plotter papercrafting machine using Floriani Craft 'N Cut software to create the input file.

The Husqvarna Epic embroidery machine

Floriani Total Control software



The Brother ScanNCut SDX225 cutter/scorer/plotter

An Embroidered Hyperbolic Fish Pattern

Previously Dunham had been able to replicate M.C. Escher's hyperbolic woodcut *Circle Limit III*, a pattern of fish using 4 colors.

Circle Limit III was based on the regular tessellation $\{8, 3\}$ of the hyperbolic plane where $\{p, q\}$ is the Schläfli symbol denoting the tessellation by regular p-sided polygons, or *p-gons*, meeting q at a vertex.

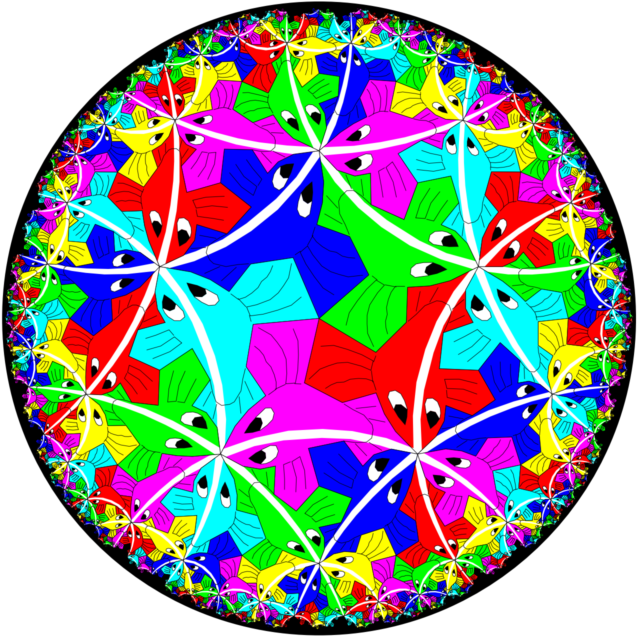
Dunham later designed a similar *Five Fish* pattern based on the $\{10, 3\}$ tessellation which required 6 colors to achieve perfect color symmetry.

After seeing some hand embroidery, Dunham guessed that his *Five Fish* pattern could be realized using embroidery techniques.

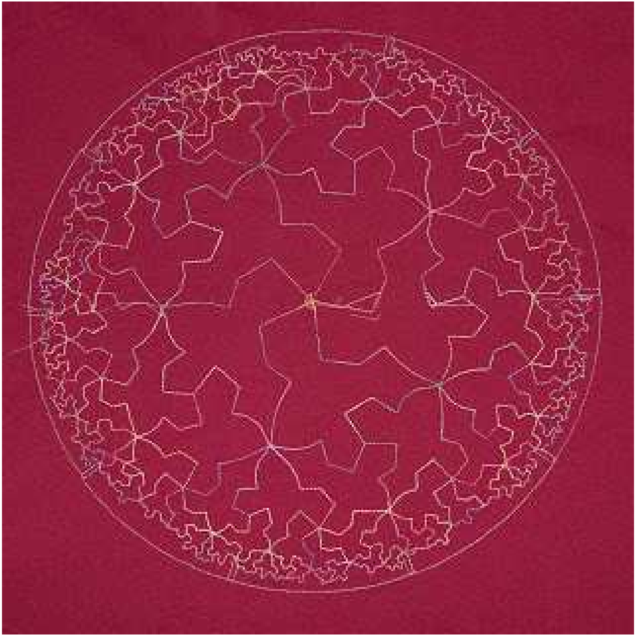
Shier provided the computer-controlled embroidery expertise needed to carry out the project, using the Husqvarna Epic embroidery machine.

M.C. Escher's woodcut Circle Limit III

Dunham’s Five Fish pattern



Aligning the fish outlines



Mostly filled fish bodies

The final embroidered Five Fish pattern

A Papercrafted Hyperbolic Pattern of Shells

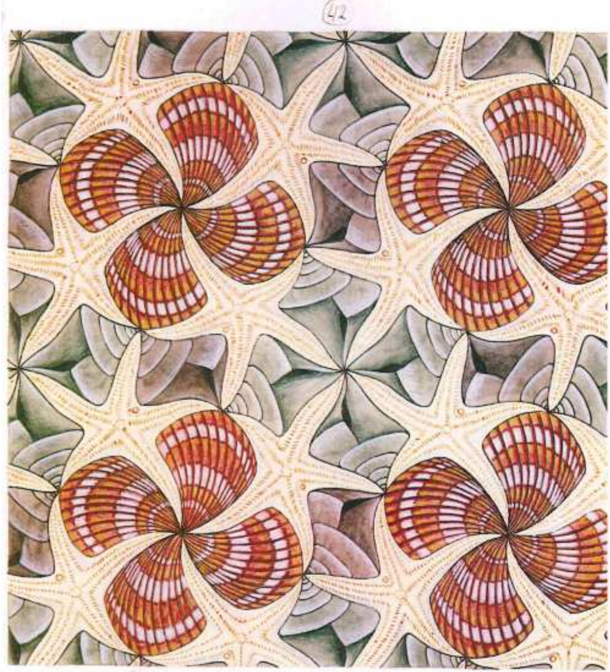
In 1941 Escher drew a pattern of shells that had symmetry group $p4$ or 442 in orbifold notation.

In about 1990, Dunham designed a related hyperbolic pattern with symmetry group 552 (in orbifold notation) based on the $\{5, 5\}$ tessellation.

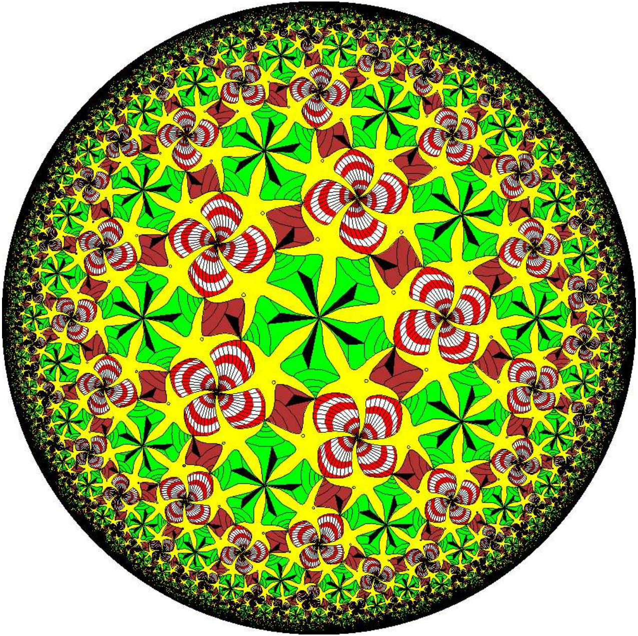
In 2019 Shier decided to implement that pattern using papercrafting, using the ScanNCut SDX225 cutter/scorer/plotter.

She made a couple of changes to Dunham's pattern. First she changed the colors of the "background" starfish to sea-star blue, the snails to pink, and the scallops to sunset red (a slight change). Second, by cutting out foam backing shapes, she raised scallops, conchs, and snails above the background starfish surface, giving the pattern a 3-dimensional effect.

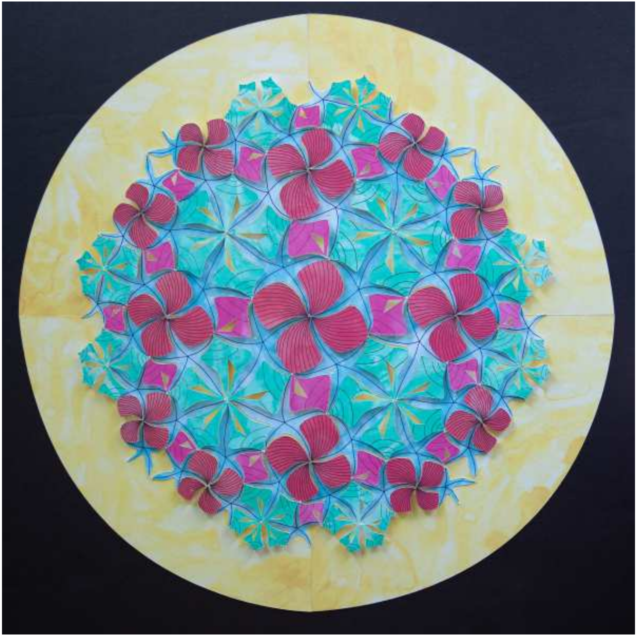
Escher's Regular Division Drawing 42



Dunham's hyperbolic 552 pattern of shells



Shier's papercrafted hyperbolic shell pattern



An oblique view showing the 3D effect

A Papercrafted Part of an Infinite Regular Polyhedron

In 1926 H.S.M. Coxeter defined *regular skew polyhedra* to be infinite polyhedra repeating in three independent directions in Euclidean 3-space.

Coxeter denoted them by the extended Schläfli symbol $\{p, q \,|\, r\}$ which denotes the polyhedron composed of p-gons meeting q at each vertex, with regular r-sided polygonal holes.

Coxeter and John Flinders Petrie proved that there are exactly three of them: $\{4, 6 \,|\, 4\}$, $\{6, 4 \,|\, 4\}$, and $\{6, 6 \,|\, 3\}$.

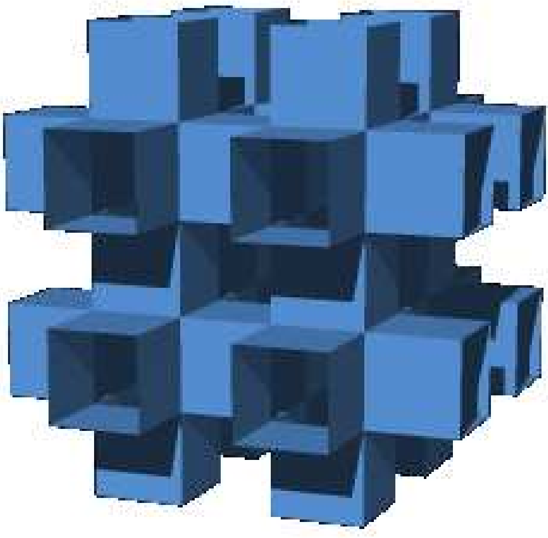
Since the sum of the vertex angles is greater than 2π, they are considered to be the hyperbolic analogs of the Platonic solids and the regular Euclidean tessellations $\{3, 6\}$, $\{4, 4\}$, and $\{6, 3\}$

In 2012 Dunham was the first person to decorate those solids with Escher-inspired patterns.

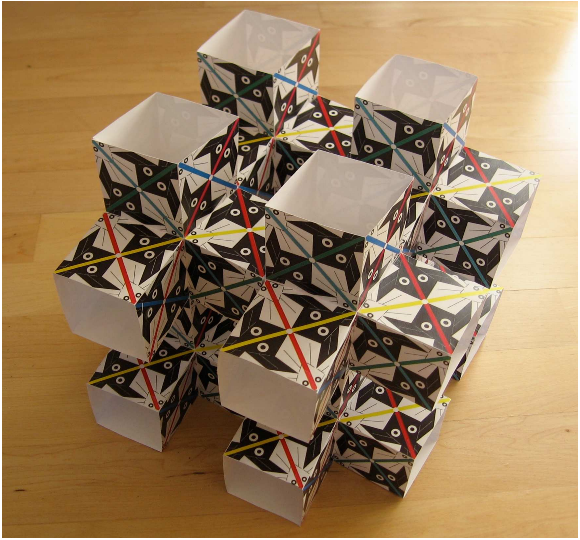
One of these, a patterned $\{4, 6 \,|\, 4\}$ polyhedron, had some flaws that Dunham thought could be fixed by re-implementing it using papercrafting, which Shier did using the ScanNCut SDX225.

The simplest regular skew polyhedron: $\{4, 6 \,|\, 4\}$

Also called the *Mucube*. It consists of invisible "hub" cubes connected by "strut" cubes, hollow cubical cylinders with their open ends connecting neighboring hubs.



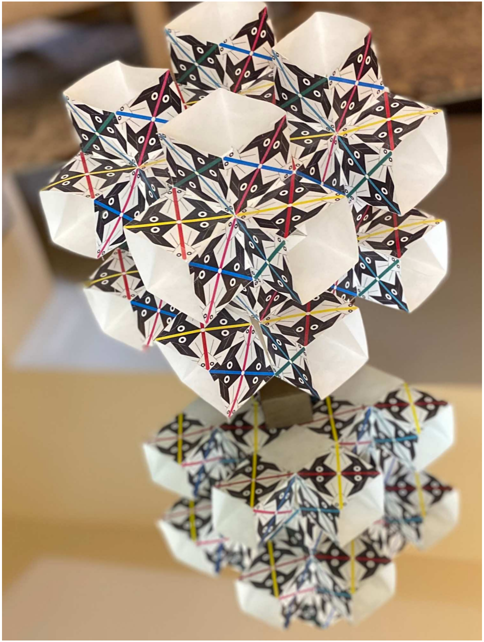
Dunham's patterned $\{4, 6 \,|\, 4\}$ with fish



Problems with Dunham's fish polyhedron

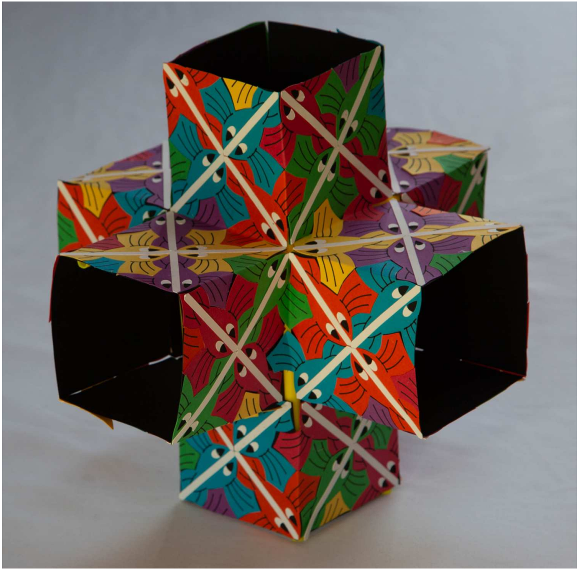
1. The fish were not consistently colored along backbone lines — they alternated from black to white and back every two fish lengths.
2. The fish also changed direction every two fish lengths — thus there was no "traffic flow" (Escher's words) in a single direction along the backbone lines.
3. The fish are very angular and not "fish-like"
4. The backbone lines of a particular color are not parallel — as can be seen in a mirror.

Dunham's fish polyhedron on a mirror



Shier's new implementation

Fixes the first and third problems.



Shier's new polyhedron on a mirror

Fixes the fourth problem.

An Embroidered and Papercrafted Dodecahedron

Shier had the concept of using the Husqvarna Epic embroidery machine to create a dodecahedron out of free-standing lace.

Each face was to be decorated with one of the capital letters of the Greek alphabet with mathematical significance.

The color of the letters would be one of red, orange, yellow, green, blue and purple evenly spaced around the color wheel, and the background would be offset by one color either direction from the letter color, for a total of 12 combinations.

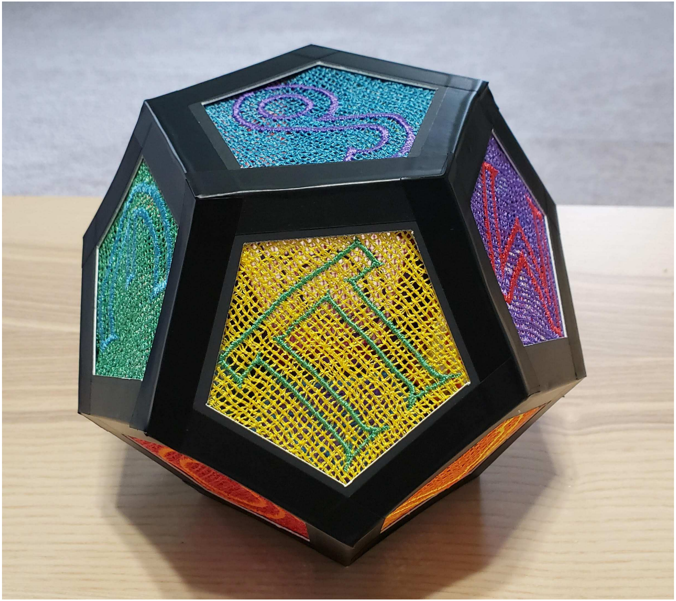
It turned out that the free-standing lace was not stiff enough to make a good looking dodecahedron, so Shier used the ScanNCut SDX225 to (1) cut posterboard pentagons to back the faces, and (2) cut black vinyl with a sticky back to hold the pentagons together.

A lace pentagon with Π

An arrangement of the 12 pentagons

The free standing lace dodecahedron

The free standing lace dodecahedron with bracing



Future Work

- There are many more hyperbolic circle patterns that could be embroidered, some more easily than others.
- And, inspired by fiber artists who have worked in 3D, we would like to embroider Escher-like patterns on 3D surfaces.
- We would also like to explore papercrafting 2D hyperbolic patterns and patterns on triply repeating polyhedra such as the regular skew polyhedra.
- Specifically, we would like to try creating a fish pattern on the $\{6, 6 \,|\, 3\}$ polyhedron which could also fix the second problem with Dunham's $\{4, 6 \,|\, 4\}$ polyhedron — so the fish all go the same direction along a backbone line.

Acknowledgements and Contact

We would sincerely like to thank all the Symmetry Festival 2021 organizers!

Contact Information:

Doug Dunham
Email: ddunham@d.umn.edu
Web: http://www.d.umn.edu/~ddunham

Lisa Shier
Email: kwajshier@yahoo.com
Blog: “Fun with a Sewing Machine”
http://funwithasewingmachine.blogspot.com/