Bridges 2010

Hyperbolic Vasarely Patterns

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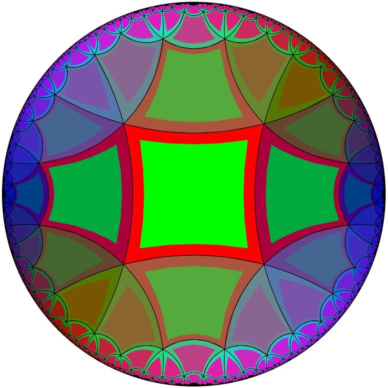


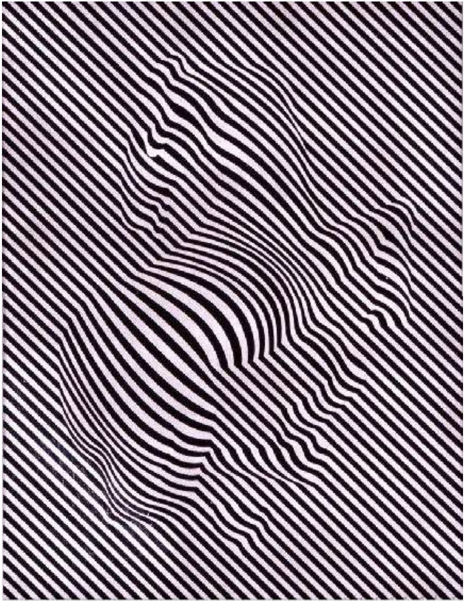
Outline

- A brief biography of Vasarely
- Hyperbolic geometry and regular tessellations
- Squares and circles on regular grids
- Randomly colored squares and circles
- Patterns based on a hexagon grid
- Future research

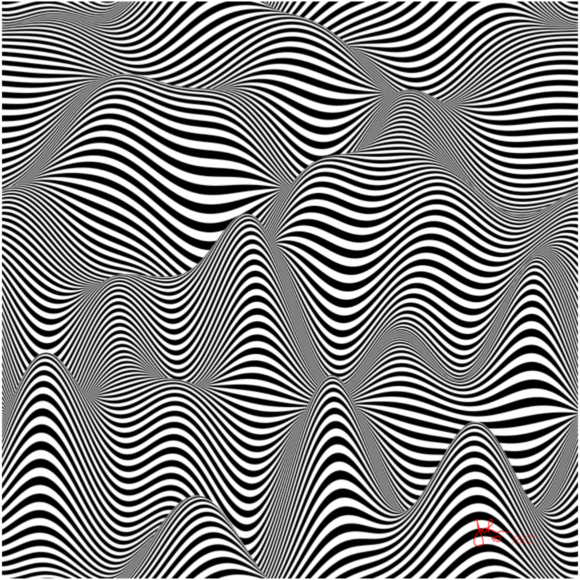
Brief Vasarely Biography

- Victor Vasarely was born Vásárhelyi Győző in Pécs April 9, 1906.
- In 1927 he abandoned medical studies and took up the study of painting.
- He moved to Paris in 1930 and spent much of his life there.
- In 1937 he created *Zebras*, generally considered to be one of the first pieces of Op Art.
- Starting in the late 1940's, he developed more geometrically abstract art.
- After a very productive and influential career, he died in Paris March 15, 1997 at age 90.

Vasarely's Zebras (1937)



John Shier's Wave (2010)



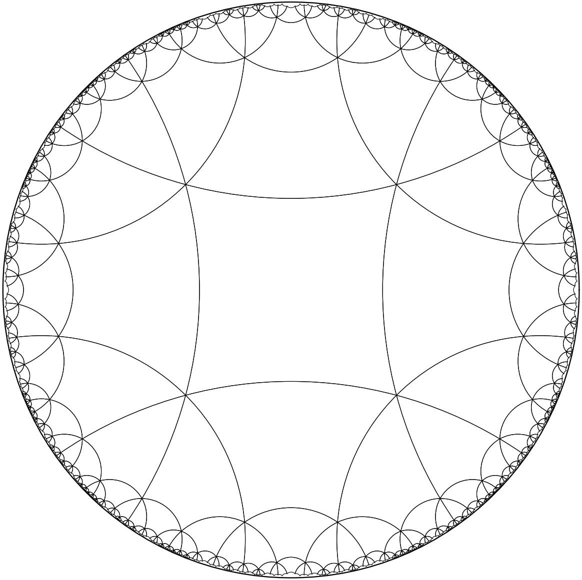
Hyperbolic Geometry and Regular Tessellations

- In 1901, David Hilbert proved that, unlike the sphere, there was no isometric (distance-preserving) embedding of the hyperbolic plane into ordinary Euclidean 3-space.
- Thus we must use *models* of hyperbolic geometry in which Euclidean objects have hyperbolic meaning, and which must distort distance.
- One such model is the *Poincaré disk model*. The hyperbolic points in this model are represented by interior point of a Euclidean circle — the *bounding circle*. The hyperbolic lines are represented by (internal) circular arcs that are perpendicular to the bounding circle (with diameters as special cases).
- This model is appealing to artests since (1) angles have their Euclidean measure (i.e. it is conformal), so that motifs of a repeating pattern retain their approximate shape as they get smaller toward the edge of the bounding circle, and (2) it can display an entire pattern in a finite area.

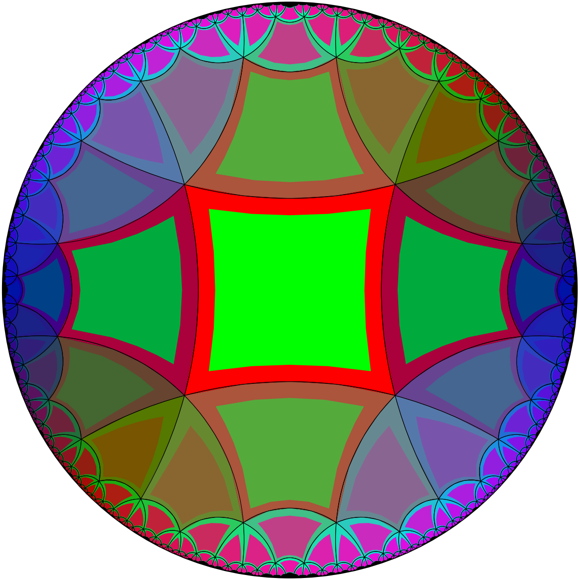
Repeating Patterns and Regular Tessellations

- A *repeating pattern* in any of the 3 "classical geometries" (Euclidean, spherical, and hyperbolic geometry) is composed of congruent copies of a basic subpattern or *motif*.
- The *regular tessellation*, $\{p, q\}$, is an important kind of repeating pattern composed of regular p-sided polygons meeting q at a vertex.
- If $(p-2)(q-2) < 4$, $\{p, q\}$ is a spherical tessellation (assuming $p > 2$ and $q > 2$ to avoid special cases).
- If $(p-2)(q-2) = 4$, $\{p, q\}$ is a Euclidean tessellation.
- If $(p-2)(q-2) > 4$, $\{p, q\}$ is a hyperbolic tessellation. The next slide shows the $\{6, 4\}$ tessellation.
- Escher based his 4 "Circle Limit" patterns, and many of his spherical and Euclidean patterns on regular tessellations.

The Regular Tessellation $\{6, 4\}$ Underlying the Title Slide Image



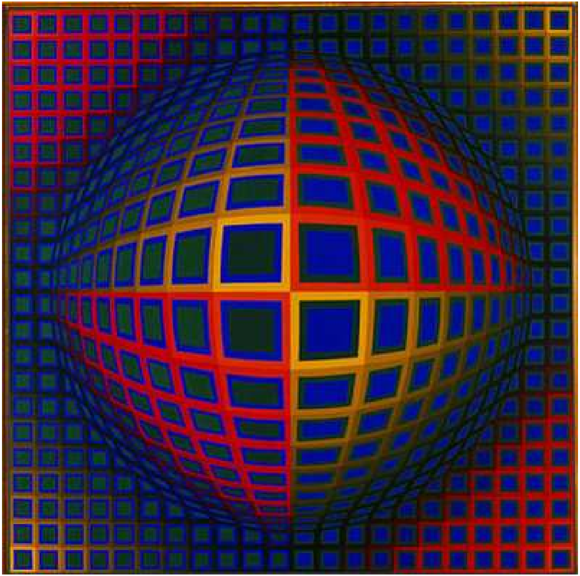
The Title Slide Pattern
(based on the $\{4, 6\}$ tessellation)



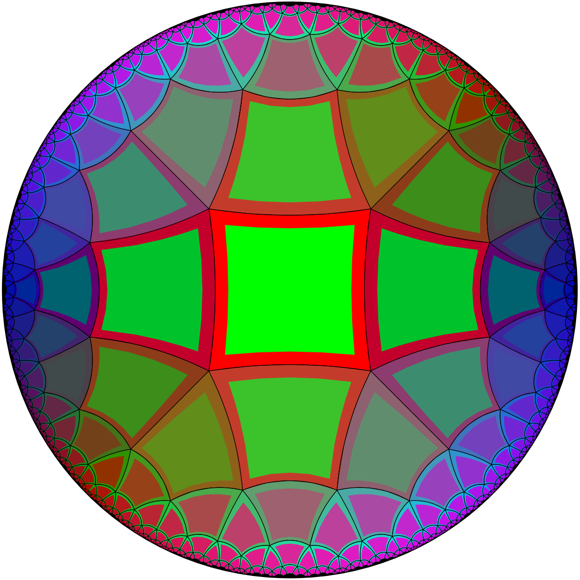
Squares and Circles on Regular Grids

- Vasarely created many patterns on square grids — i.e. $\{4, 4\}$ tessellations — filling the squares with circles or smaller squares.
- Vasarely distorted the grids in his *Vega* patterns, often producing a 3-D effect of hemispheres under the grid.
- Note that a (distorted) grid is needed to show the 3-D effect, much as a repeating pattern is needed to show the hyperbolic nature of an image in the Poincaré disk.
- In our patterns we do not distort the grids other than what is inherent in the Poincaré model.

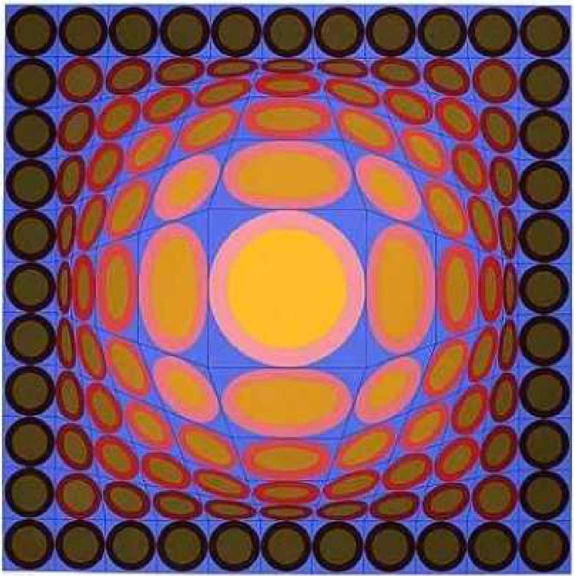
A Vasarely Vega Pattern of Squares



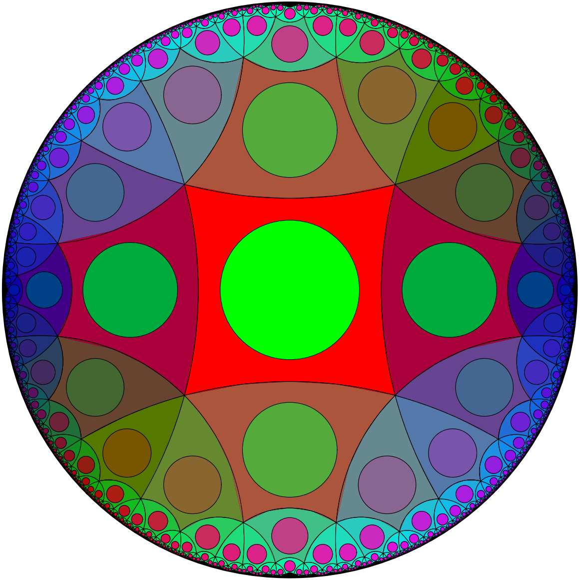
A Pattern of Squares Based on the $\{4, 5\}$ Tessellation)



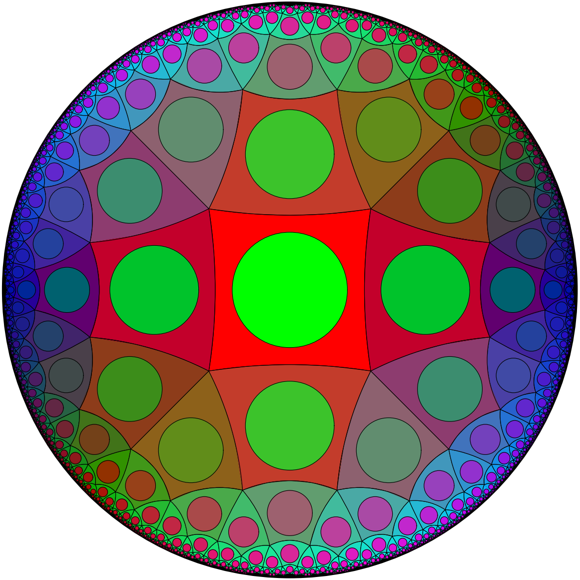
A Vasarely Vega Pattern of Circles



A Hyperbolic Pattern of Circles Based on the $\{4,6\}$ Tessellation)



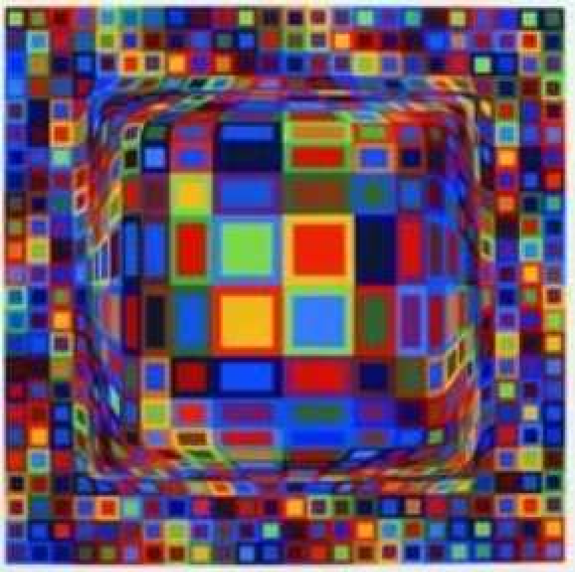
A Hyperbolic Pattern of Circles Based on the $\{4, 5\}$ Tessellation)



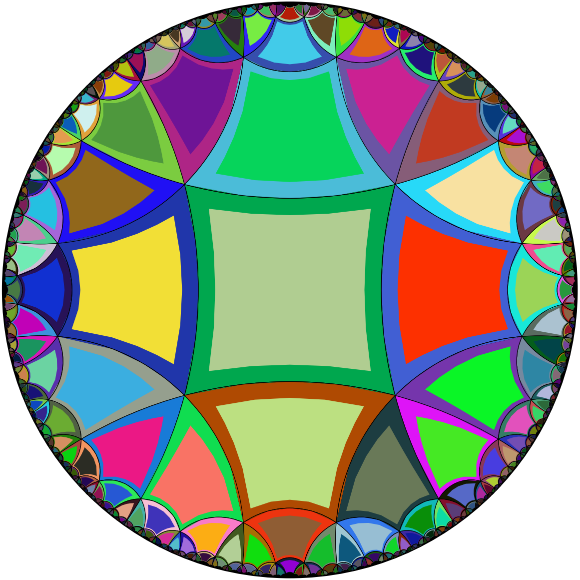
Randomly Colored Square and Circle Patterns

- Vasarely sometimes used seemingly random colors for the small squares of his patterns, however I think he chose the different colors carefully.
- Using completely random colors for squares and their frames in patterns did not seem to produce good results. However, it seemed to work better for patterns with circles within squares.
- Using a lighter version of the square color for its frame seemed to work better for square patterns.
- Vasarely always seemed to use smooth coloring for his circle patterns. In contrast, we show some randomly colored hyperbolic circle patterns.

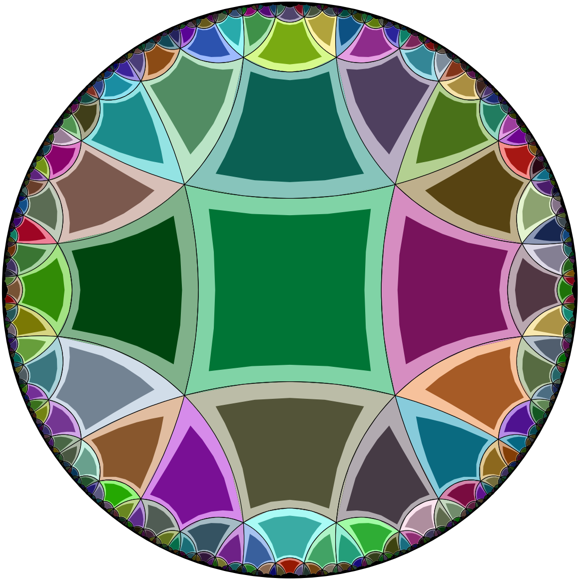
A Vasarely Pattern of “Randomly” Colored Squares



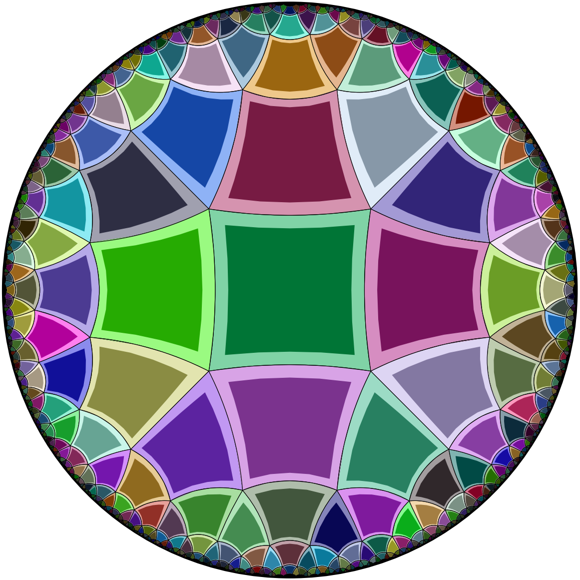
A Hyperbolic Pattern of Randomly Colored Squares with Randomly Colored Frames



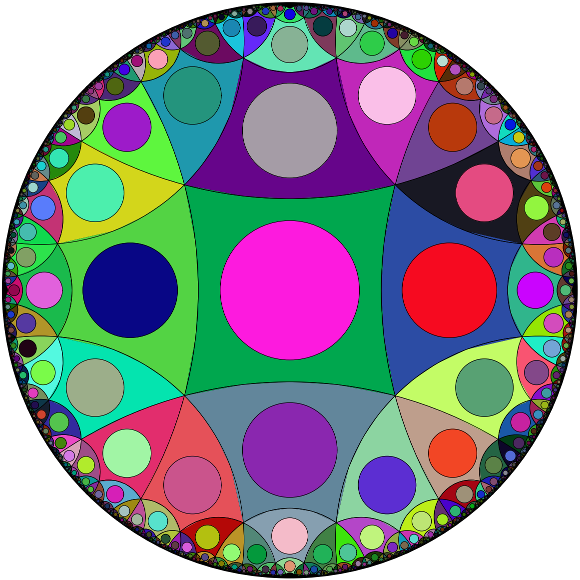
A Hyperbolic Pattern of Randomly Colored Squares with Lighter Colored Frames (a $\{4, 6\}$ pattern)



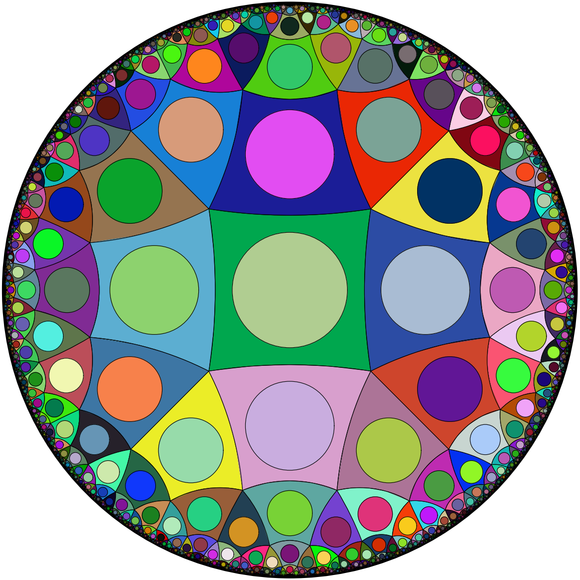
A Hyperbolic Pattern of Randomly Colored Squares with Lighter Colored Frames (a $\{4, 5\}$ pattern)



A Hyperbolic Pattern of Randomly Colored Circles with Randomly Colored Frames (a $\{4, 6\}$ pattern)



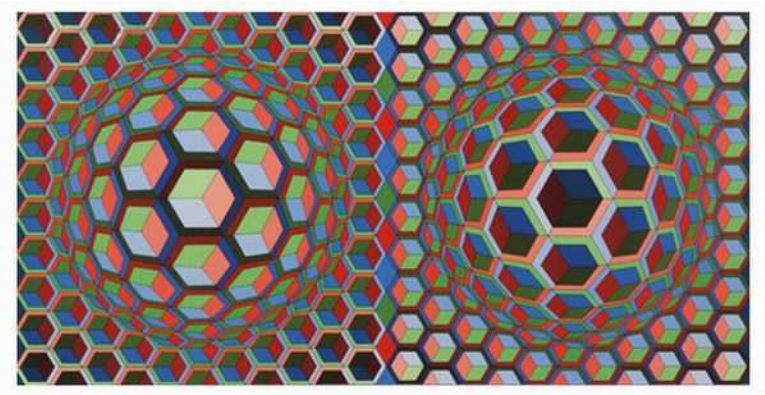
A Hyperbolic Pattern of Randomly Colored Circles with Randomly Colored Frames (a $\{4, 5\}$ pattern)



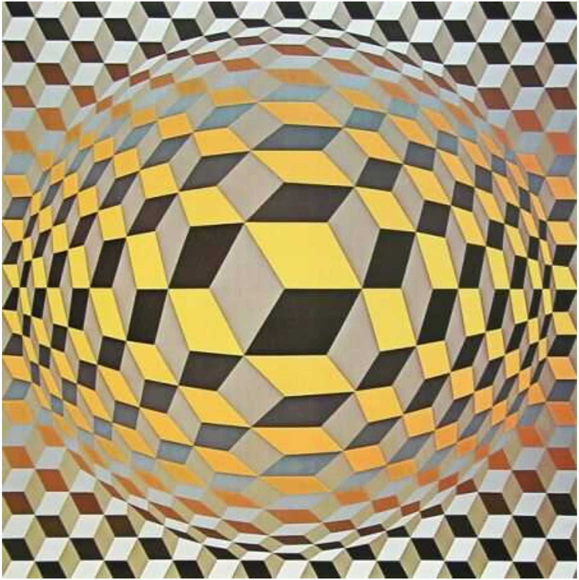
Patterns Based on a Hexagon Grid

- Vasarely made extensive use of the hexagon grid $\{6, 3\}$.
- Each hexagon was composed of 3 rhombi, colored in such a way as suggest the isometric projection of a 3D cube.
- Many of Vasarely's hexagon patterns suggested the Necker Cube phenomenon.

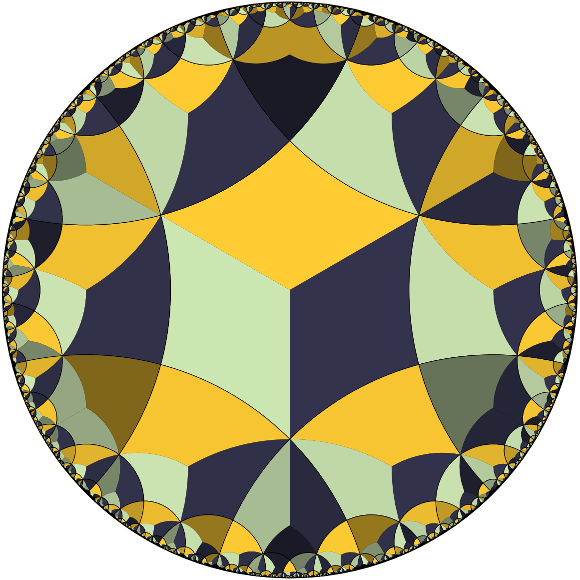
A Vasarely Hexagon Grid Pattern



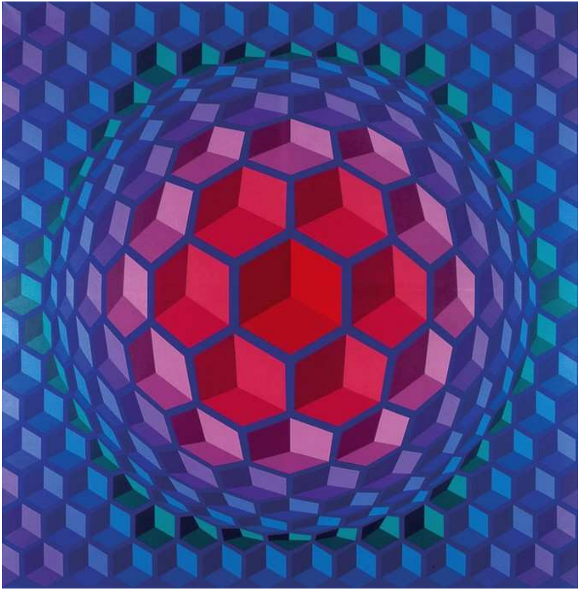
A Simpler Vasarely Hexagon Grid Pattern



A Hyperbolic Hexagon Pattern Based on $\{6, 4\}$.



A Vasarely Hexagon Grid Pattern for Future Work



References

- A link to this talk:
  `http://www.d.umn.edu/ ddunham/dunbr10tlk.pdf`
- John Shier's web site:
  `http://www.john-art.com/`
- Official Vasarely web site:
  `http://www.vasarely.com/`
- Web site showing Vasarely's *Zebras*:
  `http://artclassesva.com/?m=200907`

Future Work

- Experiment with contrasting, but not randomly colored frames for square patterns.
- Experiment with smoothly varying colorings for circle patterns.
- Try to find a better hyperbolic hexagon pattern — perhaps related to Vasarely's red and purple pattern of the last slide.

Thank You!