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# Embroidery of a Hyperbolic Fish Pattern

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# Outline

- Inspirations
- Background
- Previous hyperbolic artworks in the fiber arts
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- Our final pattern
- Another Escher-inspired hyperbolic pattern
- Conclusions and future work
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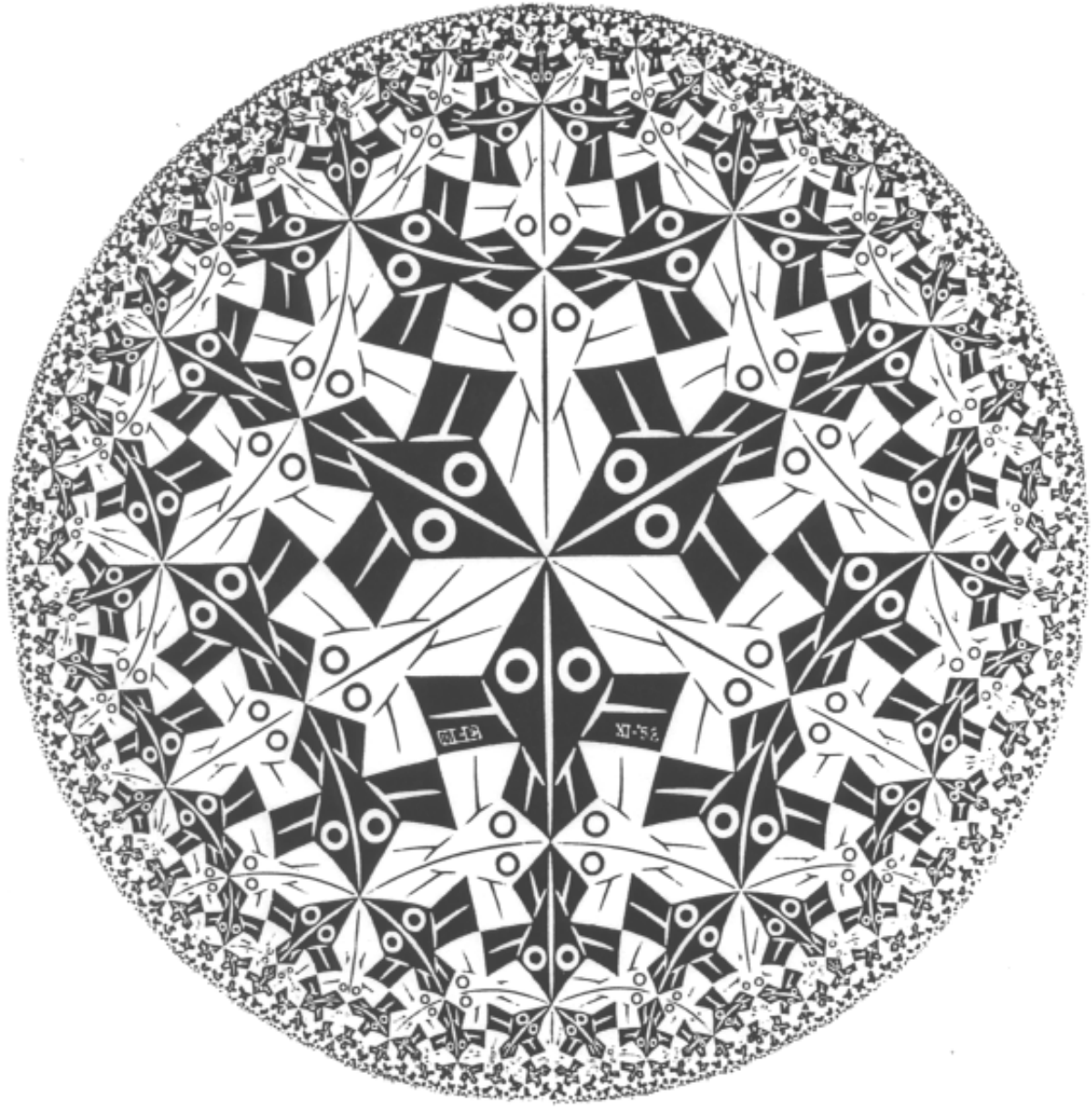
# Inspirations

Our first inspiration was M.C. Escher's four "Circle Limit" patterns, which motivated the first author, Dunham, to create a computer program that could replicate those patterns in 1980.

Secondly, in March, 2018 Dunham visited a weaving and embroidery shop in Varanasi, India and wondered if embroidery techniques could also create "Circle Limit" patterns.

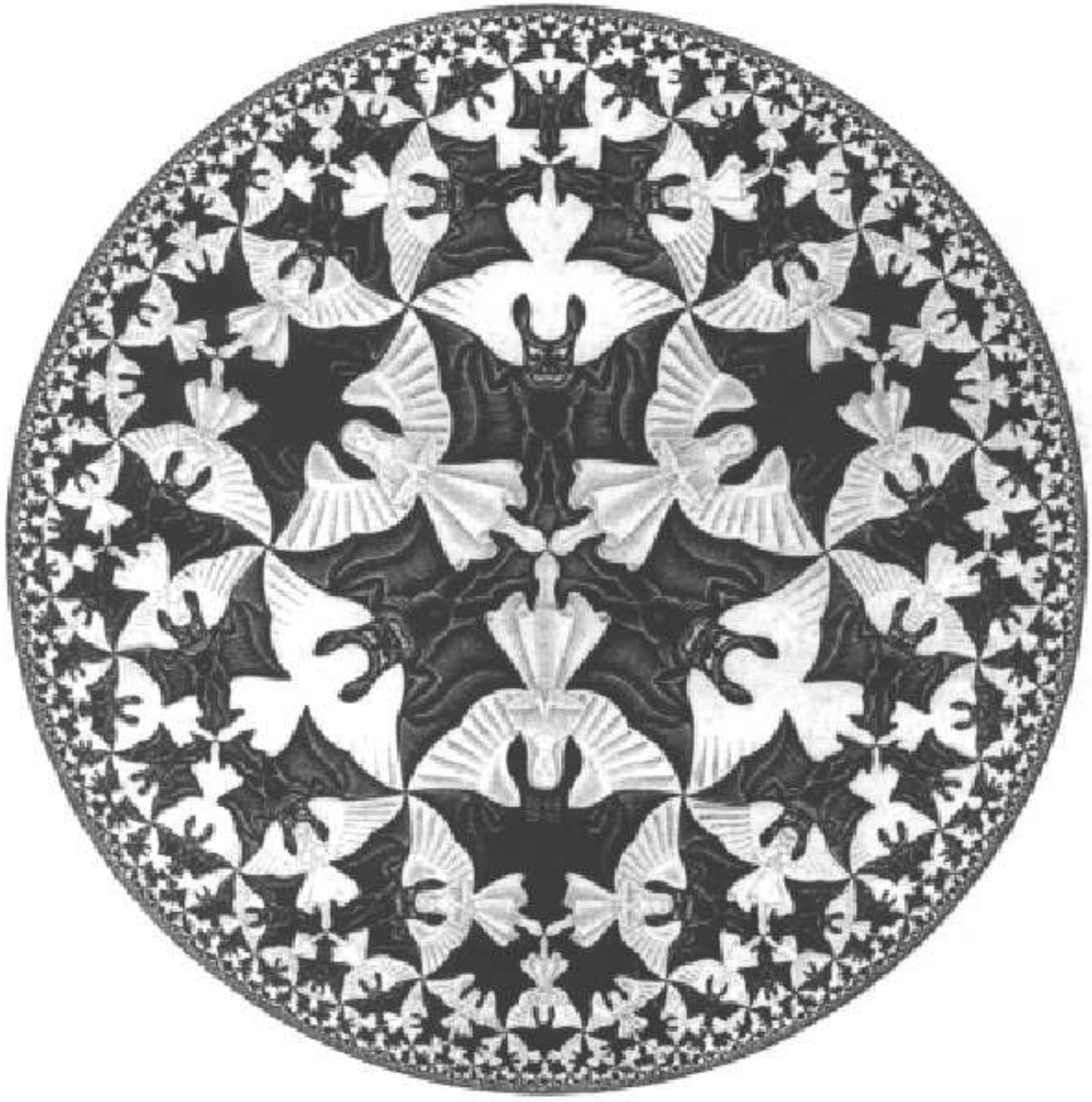
# M.C. Escher's woodcut Circle Limit I



# M.C. Escher's pattern Circle Limit II - Dunham's version

# M.C. Escher's woodcut Circle Limit III

# M.C. Escher's woodcut Circle Limit IV



# An ordinary loom in Varanasi, India

# A Jacquard loom in Varanasi, India

# Background

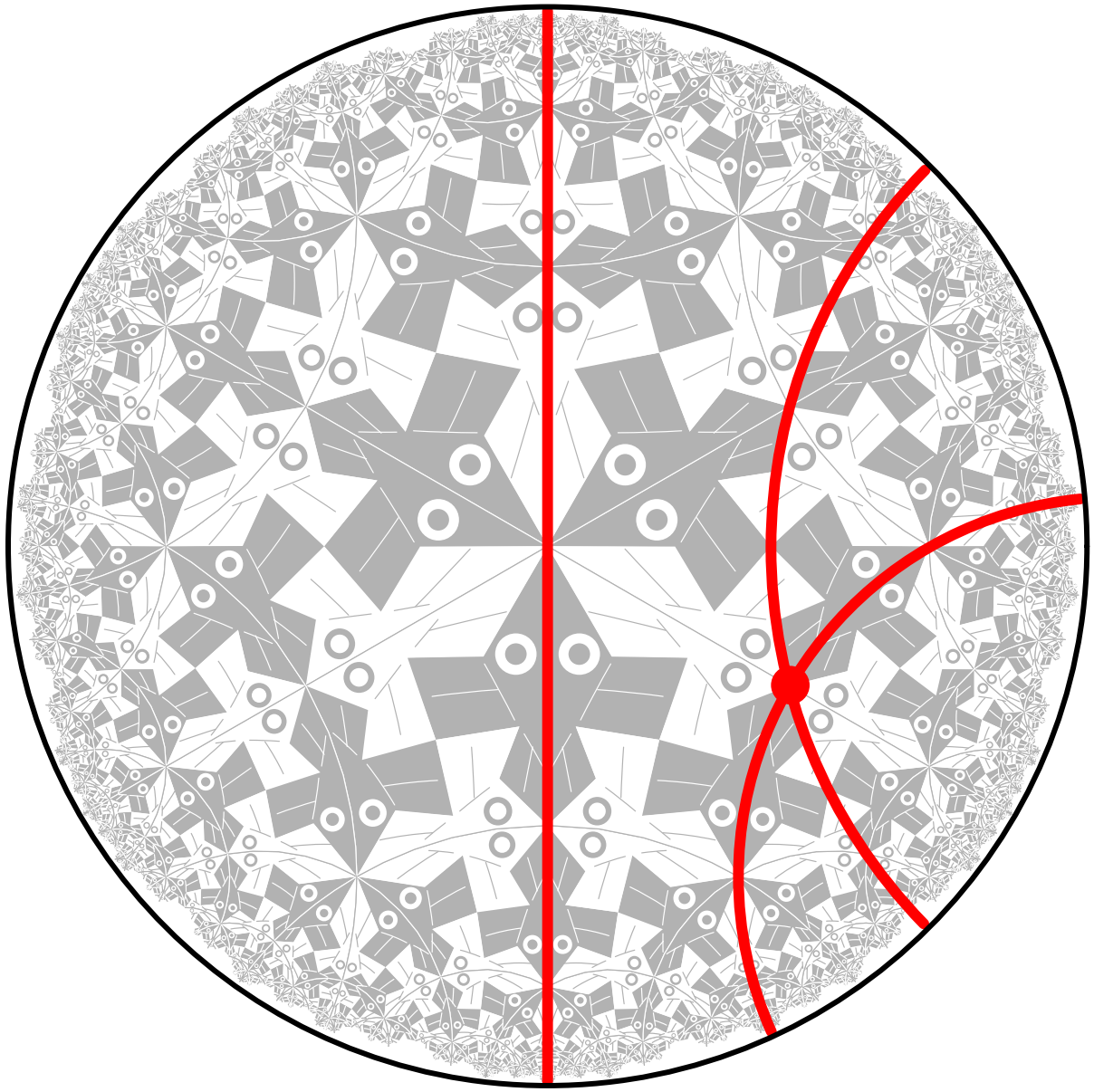
We review the following topics:

- Hyperbolic geometry.
- The hyperbolic program can create other patterns in the pattern families of which Escher's *Circle Limits* are examples, in particular the pattern of the title slide.
- A family of *Circle Limit III* patterns.

# Hyperbolic Geometry

- In 1901, David Hilbert proved that, unlike the sphere, there was no smooth isometric (distance-preserving) embedding of the hyperbolic plane into ordinary Euclidean 3-space.
- Thus we must use *models* of hyperbolic geometry in which Euclidean objects have hyperbolic meaning, and which must distort distance.
- One such model is the *Poincaré disk model*. The hyperbolic points in this model are represented by interior point of a Euclidean circle — the *bounding circle*. The hyperbolic lines are represented by (internal) circular arcs that are perpendicular to the bounding circle (with diameters as special cases).
- This model is appealing to artists since (1) angles have their Euclidean measure (i.e. it is conformal), so that motifs of a repeating pattern retain their approximate shape as they get smaller toward the edge of the bounding circle, and (2) it can display an entire pattern in a finite area.

# Poincaré Disk Model of Hyperbolic Geometry



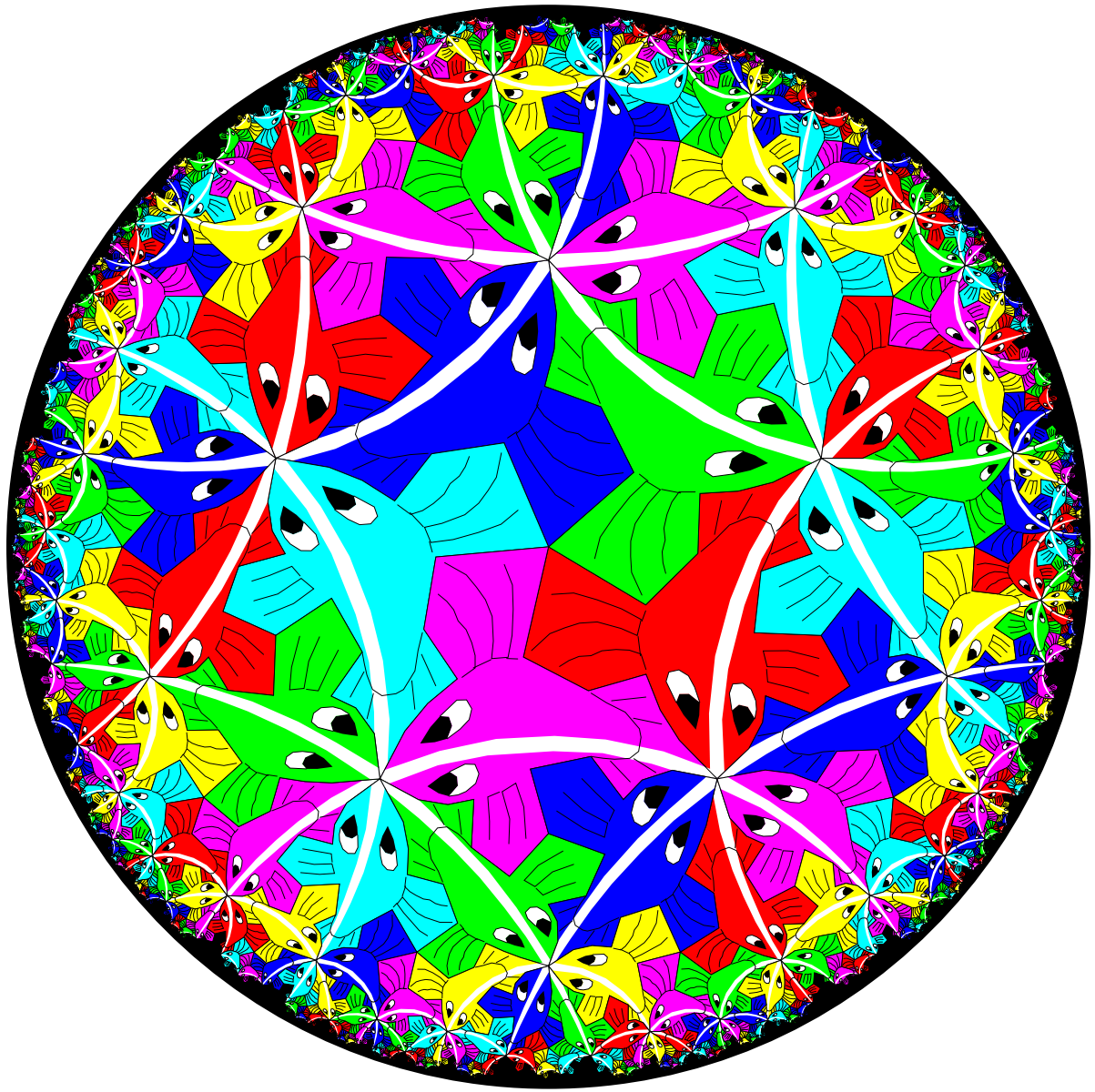
# A Family of *Circle Limit III* Patterns

We use the symbolism **(p,q,r)** to denote a pattern of fish in which **p** meet at right fin tips, **q** meet at left fin tips, and **r** fish meet at their noses. Of course **p** and **q** must be at least three, and **r** must be odd so that the fish swim head-to-tail (as they do in *Circle Limit III*).

Escher's *Circle Limit III* pattern itself would be labeled (4,3,3) in this notation.

We note that our (5,3,3) pattern of the title slide requires six colors, whereas *Circle Limit III* only needs four. In fact the color symmetry group of our pattern is the alternating group $A(5)$.

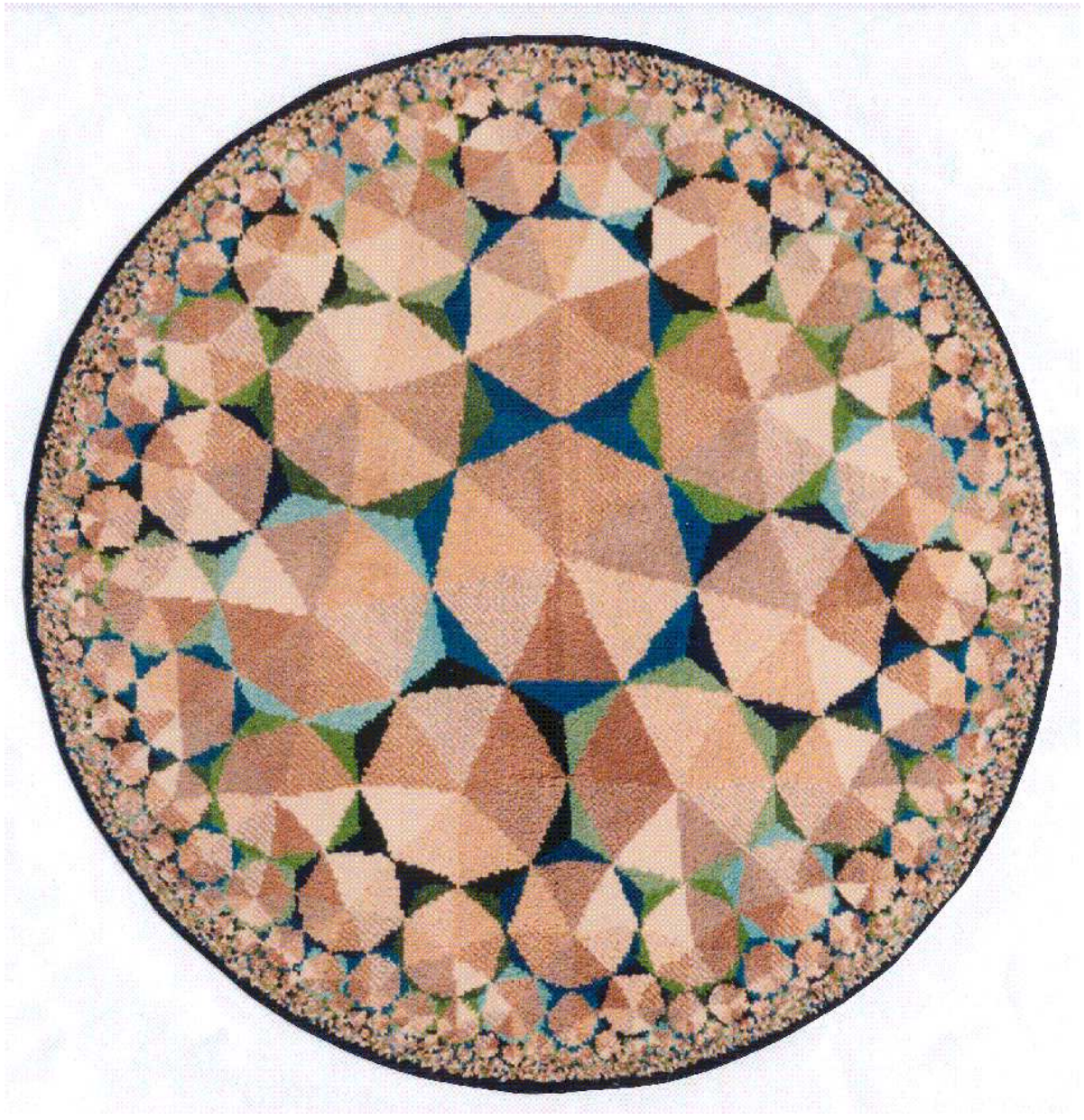
# The (5,3,3) pattern of the title slide



# Previous Hyperbolic Artworks the Fiber Arts

- 2D patterns using the Poincaré model
  - Tony Bomford's hooked rugs
  - Mary Williams' quilt
- 3D works
  - Diana Taimina's crochet works
  - S. Louise Gould's triply repeating polyhedra

# Tony Bomford's Rug 17



# Mary Williams' quilt "Poincaré"

# Diana Taimina's $\{6,4\}$ surface

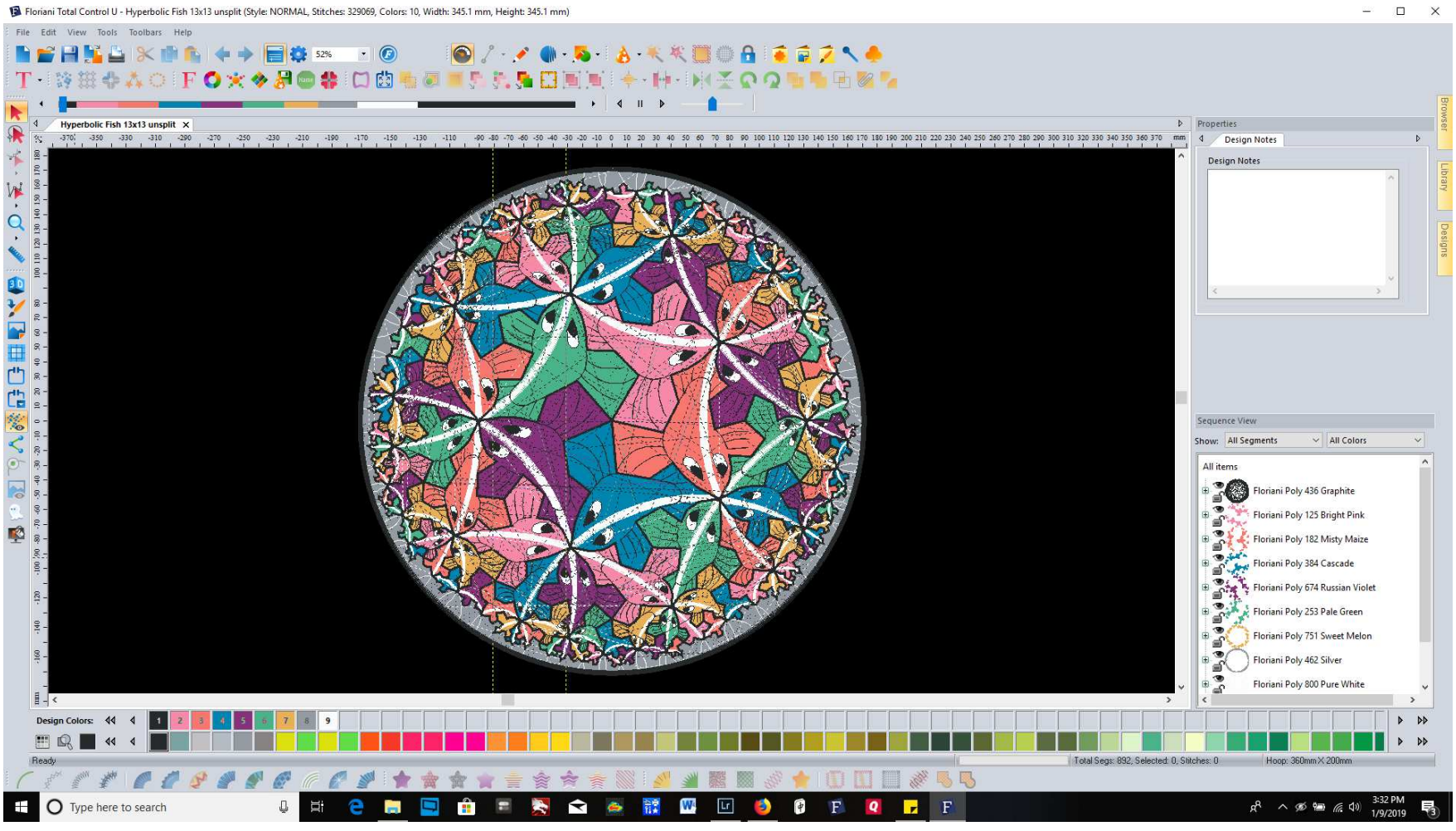
# Louise and Frank Gould's (5.5.5.5.5) surface

# Our First Pattern

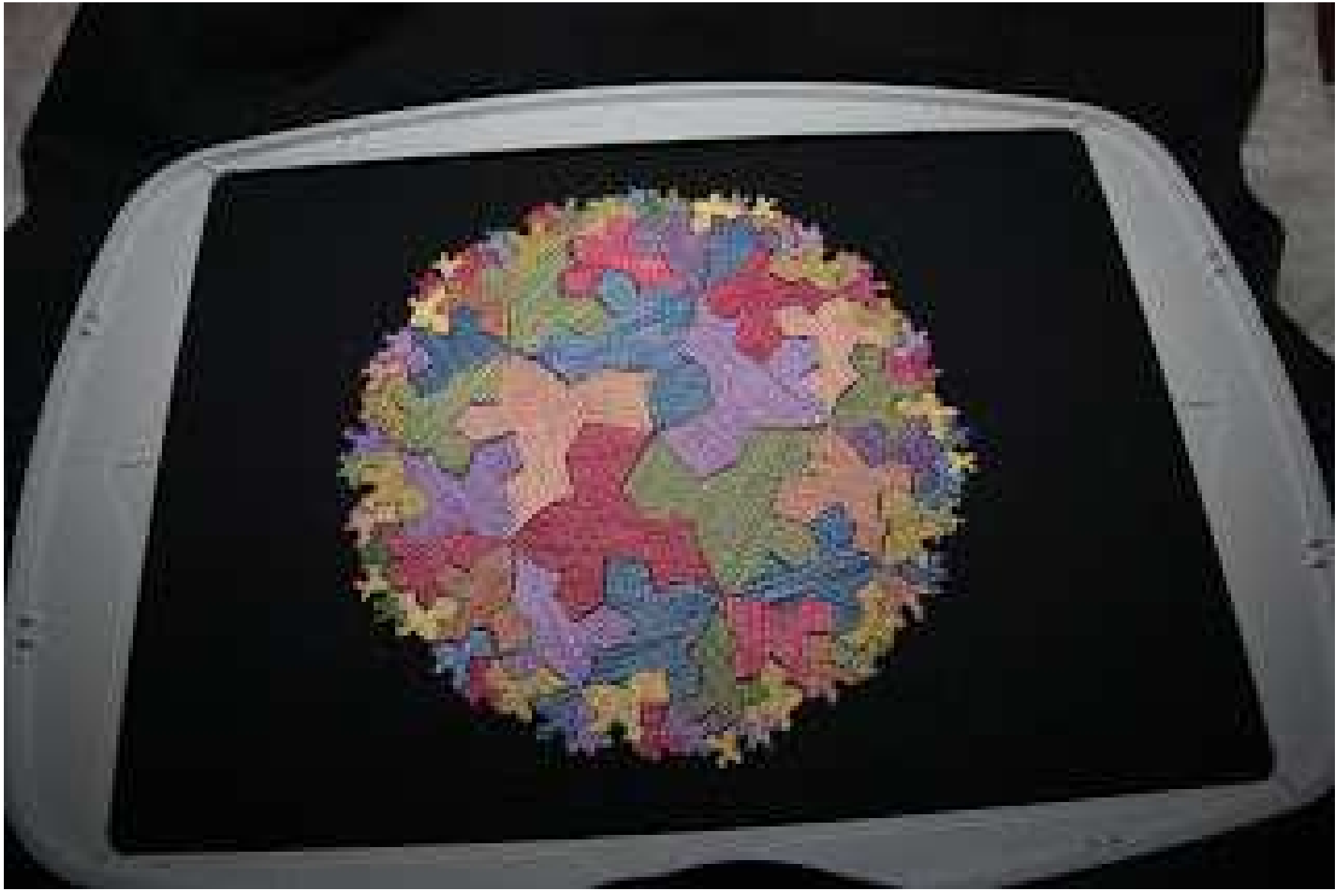
- Used a Viking Husqvarna Epic embroidery machine wth a 260mm by 360mm hoop.
- And used Floriani Total Control embroidery software.

# Husqvarna Epic embroidery machine

# Floriani Total Control embroidery software



# The pattern with only the fish bodies

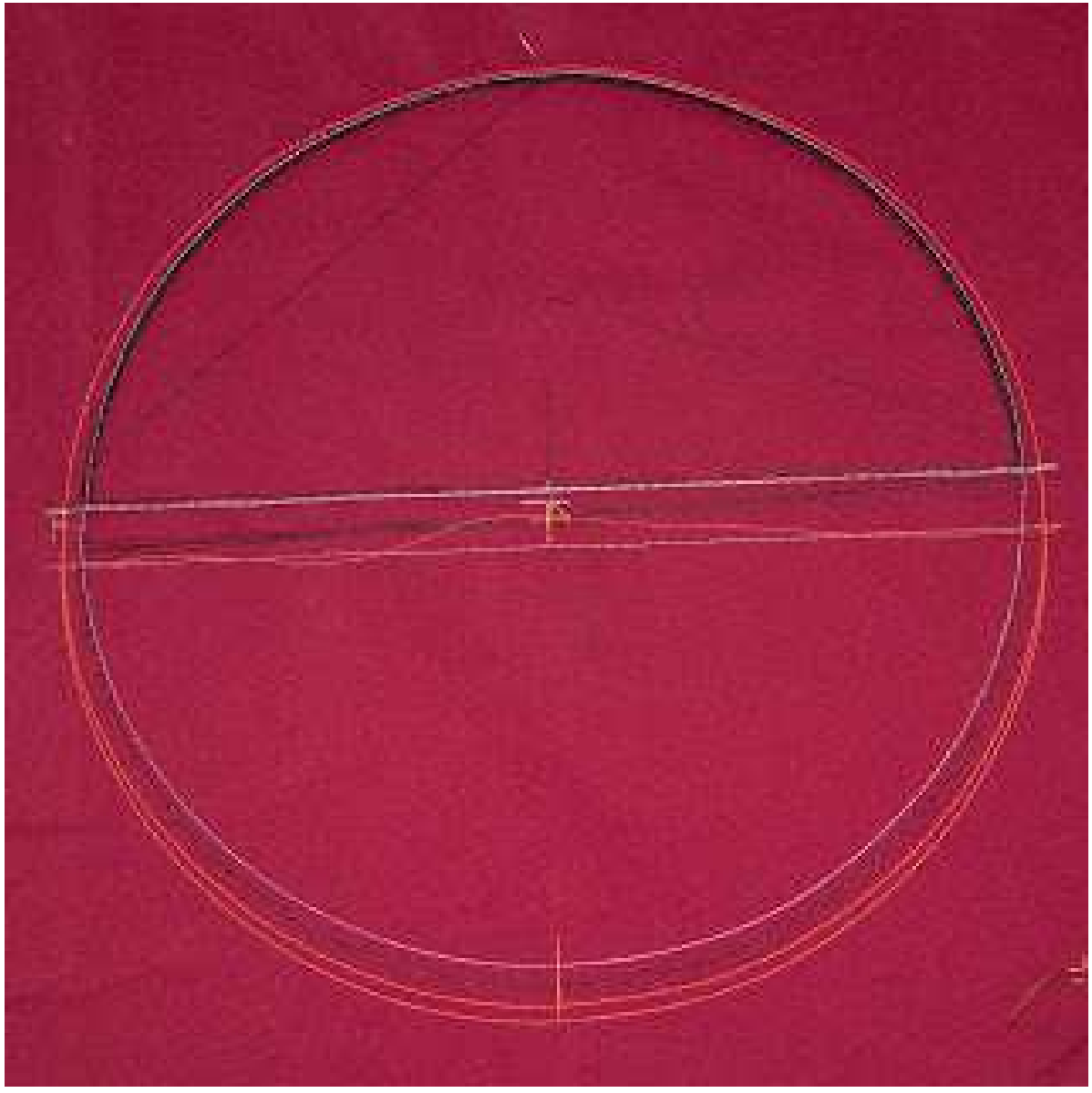


# The completed 10.5 inch pattern

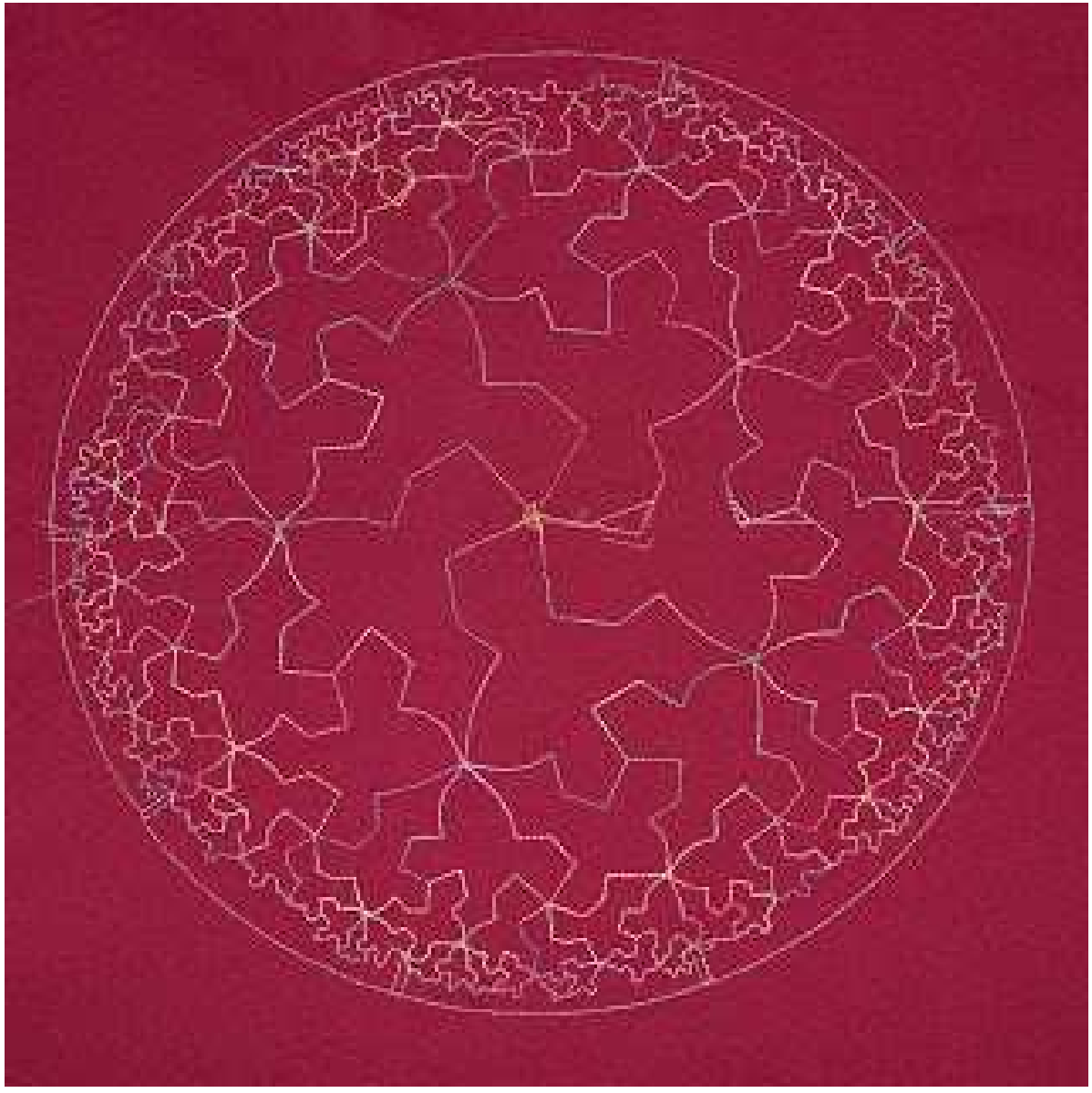
# Our Final Pattern

- Used a 350mm by 360mm Husqvarna Majestic hoop — the largest available.
- Advantages:
  - A bigger pattern! — 13.75 inches.
  - More fish — the fish pattern gets closer to the bounding circle.
  - More detail within the outer fish.
- Disadvantages:
  - It could only be used to stitch half of the pattern at a time.
  - The hoop must be rotated 180 degrees to stitch the second half — actually it had to be rotated many time to complete the pattern.
  - After rotation the hoop had to be carefully aligned.

# Aligning the bounding circle



# Aligning the fish outlines



# Partially filled pattern

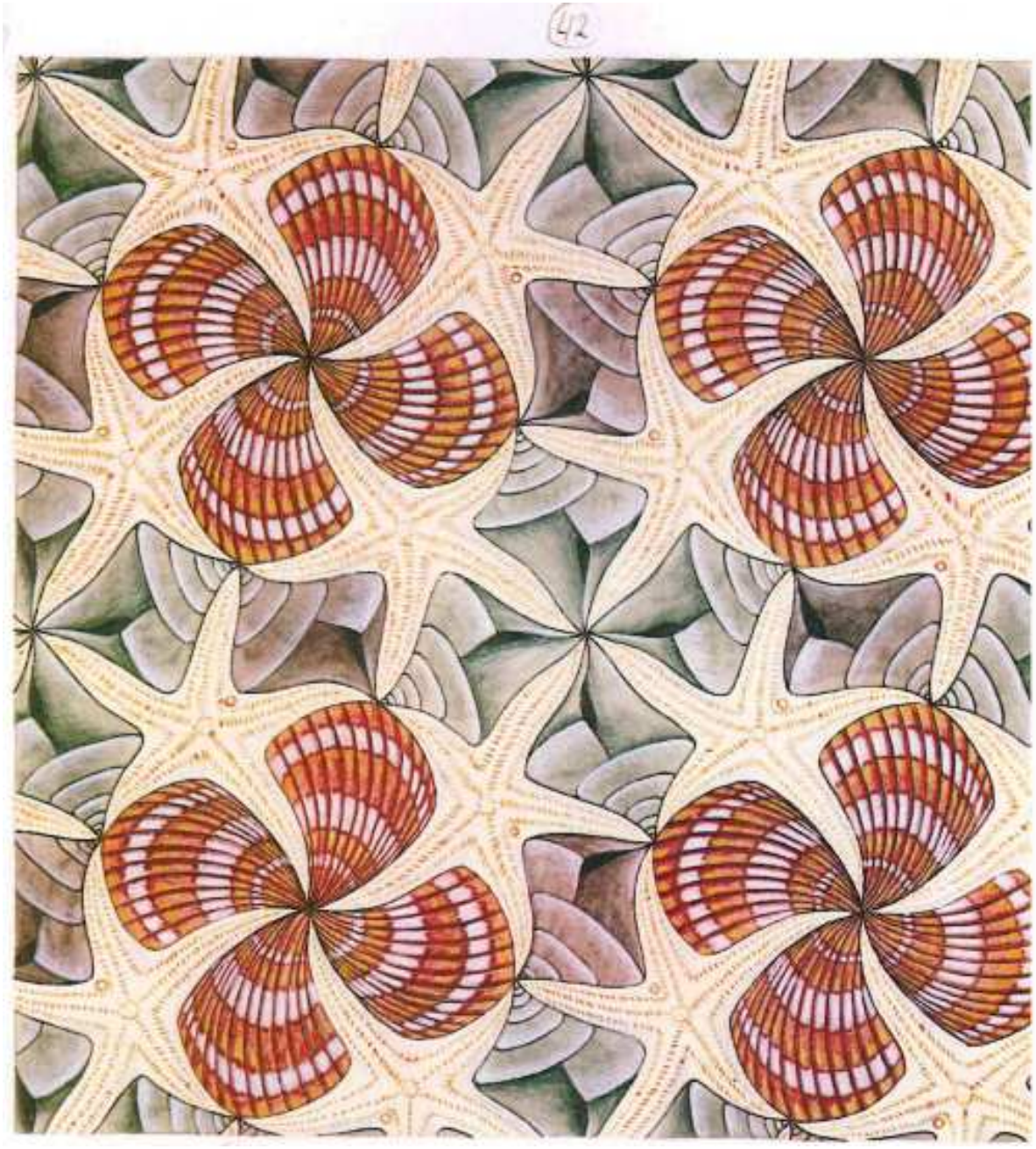
# The final pattern

# The 10.5 inch pattern for comparison

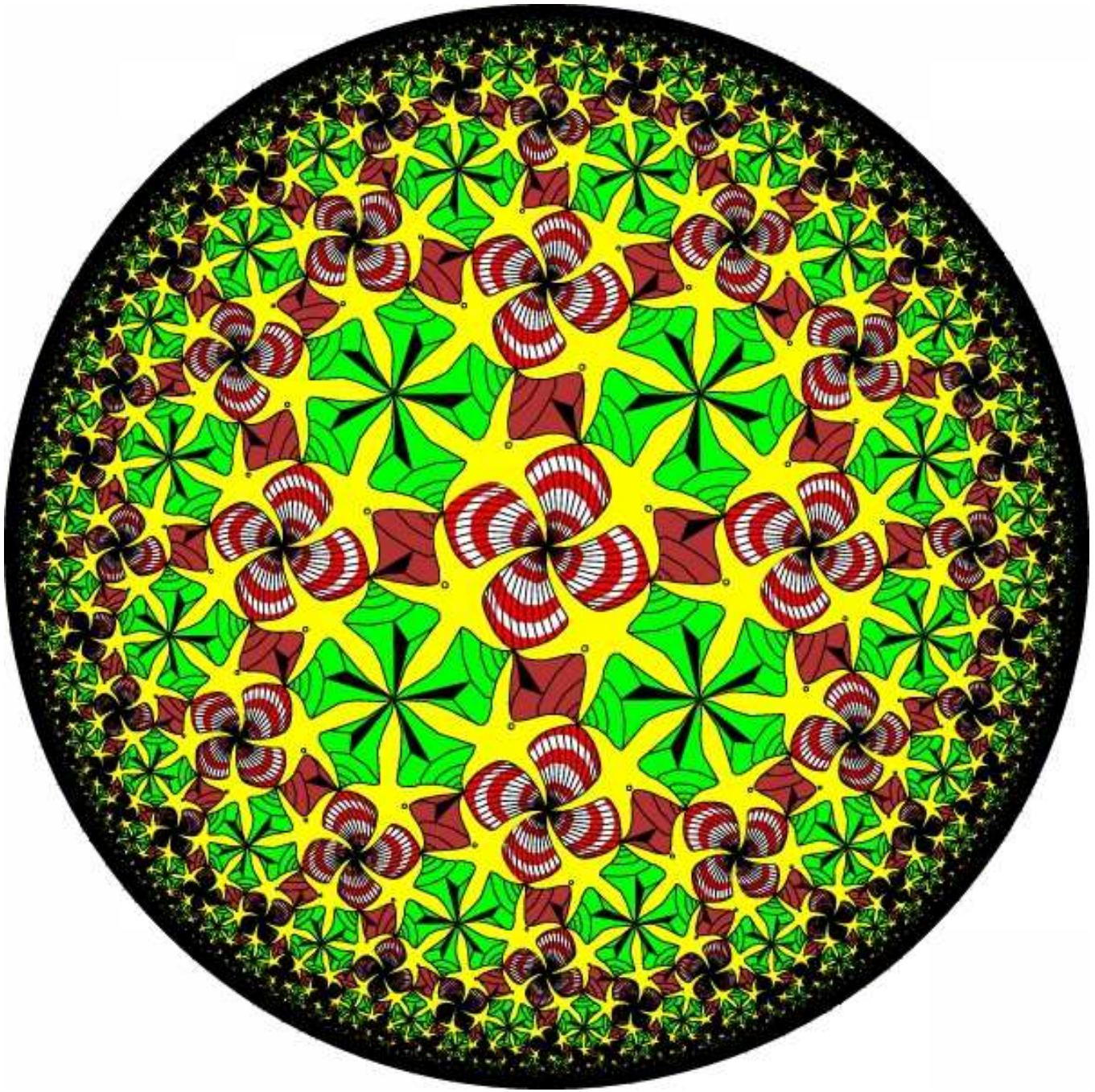
# Another Escher-inspired hyperbolic pattern

- Escher's Regular Division Drawing 42 of shells
- A related hyperbolic pattern of shells

# Escher's Regular Division Drawing 42



# A hyperbolic pattern of shells



# Future Work

- There are many more hyperbolic circle patterns that could be embroidered, some more easily than others.
- And, inspired by fiber artists who have worked in 3D, we would like to embroider Escher-like patterns on 3D surfaces.
- We would also like to explore papercrafting a hyperbolic pattern with a digital cutter/printer.
- Another possibility is patterns on free standing lace.

# Acknowledgements and Contact

We would sincerely like to thank all the Bridges 2019 organizers!

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