Tony Bomford's Hyperbolic Hooked Rugs

Douglas Dunham
Department of Computer Science
University of Minnesota, Duluth
Duluth, MN 55812-2496, USA
E-mail: `ddunham@d.umn.edu`
Web Site: `http://www.d.umn.edu/˜ddunham/`

Dedicated to the memory of Tony Bomford

Abstract

Tony Bomford made 18 hooked rugs with mathematical themes, six of them based on hyperbolic geometry. In this paper we show and analyze four of those hyperbolic rugs.

1. Introduction

In 1981 Tony Bomford was inspired to make his first hyperbolic rug *Hyperbolic Spiderweb*, Figure 1, by M. C. Escher's hyperbolic pattern *Circle Limit IV,* Figure 2, which he saw in H. S. M. Coxeter's chapter "Angels and Devils" in *The Mathematical Gardner* [Coxeter2]. *Hyperbolic Spiderweb* was Bomford's twelfth

Figure 1: Tony Bomford's Rug 12: *Hyperbolic Spiderweb.*

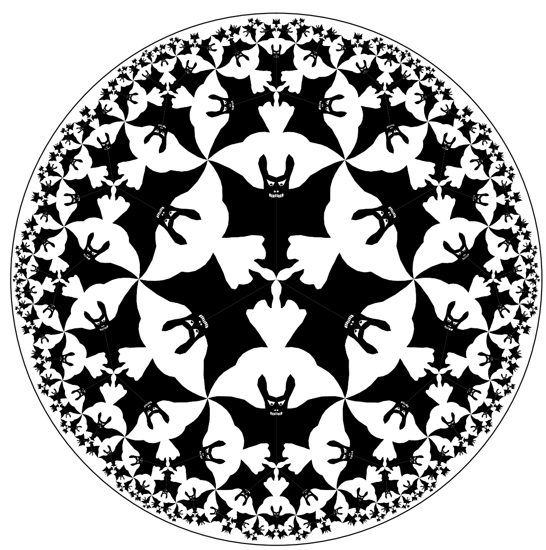

Figure 2: A computer-generated rendition of M. C. Escher's print *Circle Limit IV.*

hooked rug; the previous eleven were Euclidean. He created six more rugs — all of them hyperbolic except for rug number 14 which was based on one of Roger Penrose's non-periodic tilings of the (Euclidean) plane.

We start by giving a biographical sketch of Tony Bomford's life in Section 2. In Section 3, we analyze the rugs numbered 12 and 13, which are related. Similarly, the rugs numbered 15 and 18 are related and are discussed in Section 4. Finally, the related rugs numbered 16 and 17 are discussed in Section 5.

2. Tony Bomford

Anthony Gerald (Tony) Bomford was born to English parents in Dehra Dun, India, January 17, 1927. Tony's father, Guy, a geodesist, was an officer in the Corps of Royal Engineers attached to the Survey of India. Tony spent his pre-school years in India and then was sent to Shrewsbury School in England for his education. He joined the Royal Engineers upon graduation in 1944. The Engineers supported him for two very successful sessions at Cambridge University, plus a one-year extension to do specialized studies in mathematics. After that, he participated in surveying and mapping expeditions to Tanzania (then Tanganyika), South Georgia, and Australia. Next, Tony spent two years with the Ordnance Survey in England before returning in 1961 to Australia, which was to become his home for the rest of his life. He joined the Australian Division of National Mapping as a senior surveyor, rising in rank to become its director in 1977. Five years later in 1982, Tony took early retirement from this position to pursue his many interests, which included travel, stamp collecting, poetry, music, cutting polyhedra from red box timber, and making mathematical hooked rugs. After 20 years of retirement rich with activities, Tony passed away on May 10, 2003.

3. Rugs 12 and 13 and Hooked Rugs in General

As mentioned above, Tony Bomford's first hyperbolic rug number 12 was inspired by M. C. Escher's hyperbolic pattern *Circle Limit IV*, Figure 2. Similarly, Escher's initial inspiration for his *Circle Limit* patterns was the pattern shown in Figure 3, which he saw in a reprint of Coxeter's paper "Crystal Symmetry and Its Generalizations" [Coxeter1]. In Figure 3, twelve of the 30-45-90 degree triangles sharing a common vertex of 30-degree angles form a regular hyperbolic hexagon. These hexagons, which meet 4 at a vertex, form the regular tessellation $\{6, 4\}$ shown in Figure 4. In general the regular tessellation $\{p, q\}$ by p-sided regular

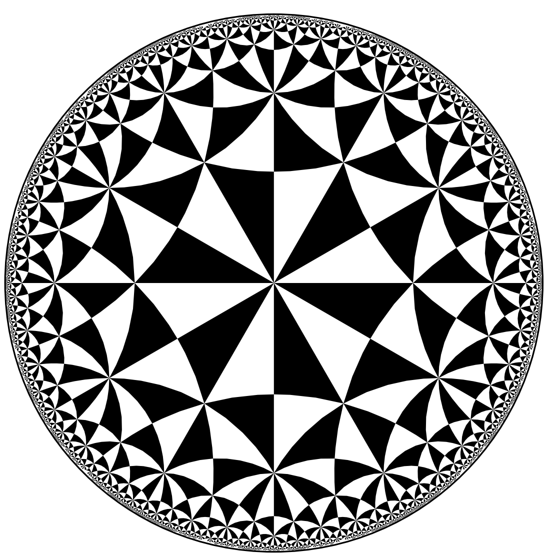

Figure 3: The pattern from Coxeter's paper that inspired M. C. Escher's *Circle Limit* patterns.

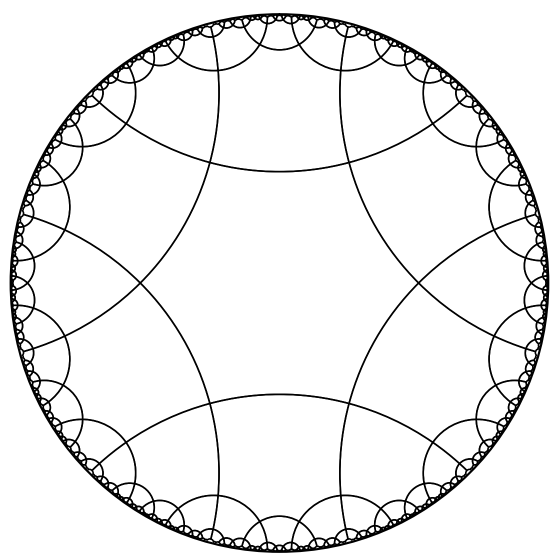

Figure 4: The regular tessellation $\{6, 4\}$

polygons meeting q at a vertex exists in hyperbolic geometry if $(p-2)(q-2) > 4$.

The radii of the hexagons of $\{6, 4\}$ divide each of the hexagons into 45-45-60 degree isosceles triangles. This is shown in Figure 5 where the radii are shown as thin lines and $\{6, 4\}$ is shown as thick lines. These isosceles triangles can clearly be seen in rug 12 — Figure 1 above. The radii are each divided into eight equal hyperbolic segments which form the ends of the curved strips of different colors in rug 12. The actual colors are white, and shades of brown, tan, and red. Bomford started rug 12 on the 23rd of December, 1981, and finished it October 19, 1982. The rug is 2.18 meters in diameter and contains 64,242 knots.

Coxeter, Escher, and Bomford all used the *Poincaré disk model* of hyperbolic geometry whose points are the interior points of a bounding circle, and whose (hyperbolic) lines are circular arcs orthogonal to the bounding circle (including diameters).

Bomford's rug 13, *Hyperbolic Lagoon*, shown in Figure 6, is also based on the tessellation $\{6, 4\}$. In

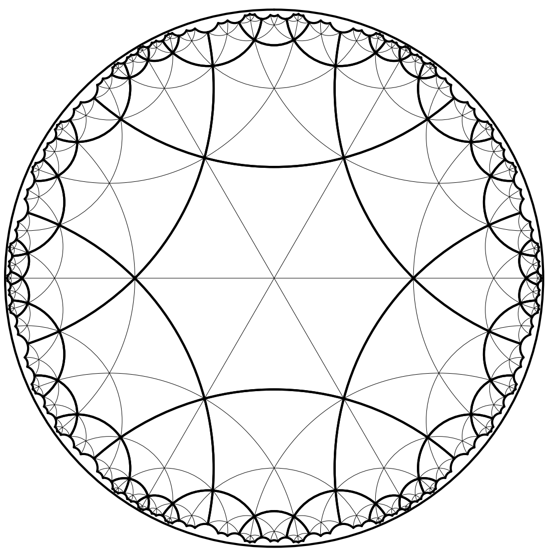

Figure 5: The radii (thin lines) of the hexagons of the tessellation $\{6, 4\}$ (bold lines).

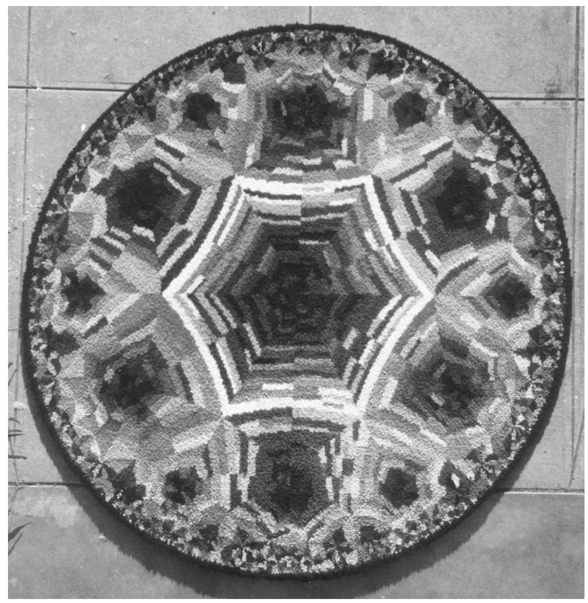

Figure 6: Bomford's rug 13 *Hyperbolic Lagoon.*

fact the same calculations for the divisions of the radii were used as in rug 12. However, unlike rug 12, the curved strips usually change color midway between the radii — i.e. at the apothems (the altitudes of the isosceles triangles, from the center of the hexagons to the midpoints of its sides). And some strips even change colors between the radii and the apothems. Rug 13 gets its name from the lagoon-related colors it uses: shades of blue, green, aqua, and tan (sand). Rug 13 was started on the 24th of October, 1982 (five days after rug 12 was finished!) and completed November 11, 1983. It is the same size as rug 12.

Hooked rugs are made by pulling loops of thin strips of colored cloth (often wool) through holes in a backing material — now usually burlap (jute Hessian cloth). Bomford used the term *knot* to denote one loop pulled through one hole. Happy and Steve DiFranza give a brief description of how to make hooked rugs at [DiFranza1]. The burlap is stretched tautly over a frame and strips of the cloth 1/8 to 1/2 inch wide are pulled up with a hooked tool (like a knitting needle with a handle) about half an inch through the grid holes in the burlap. It is not clear how long hooked rugs have been made, but according to Vicky Rumble, the history may go back more than 1500 years [Rumble1]. Tony Bomford made 18 hooked rugs, the last one being almost, but not quite finished.

4. Rugs 15 and 18

As mentioned above, Bomford took a "vacation" from hyperbolic rugs and created a non-repeating Euclidean tiling for rug 14, using Roger Penrose's kites and darts. During this time it was suggested (I think by Coxeter) that Bomford should contact me about other hyperbolic tessellations. This paralleled events about 25 years earlier when Escher asked Coxeter if there were any other regular hyperbolic tessellations besides $\{6, 4\}$. Among other materials, I sent Bomford large prints of the $\{5, 4\}$ and $\{7, 3\}$ tessellations produced by a 36-inch wide pen plotter to which I had access at the time (these plots were recently re-discovered by Richard Bomford in his father's house). I chose those two tessellations since their regular p-sided polygons (and those of their duals $\{4, 5\}$ and $\{3, 7\}$) have the smallest angle defects among all the regular hyperbolic tessellations.

The plot of the tessellation $\{5, 4\}$ was the inspiration for Rug 15, Figure 7. It is based on the semi-regular, or more accurately *uniform*, tiling $(4.5.4.5)$, in which two squares and and two pentagons alternate around each vertex. Grünbaum and Shephard discuss uniform tilings in Chapter 2 of [Grünbaum1]. The $(4.5.4.5)$ tiling can be constructed from the regular tessellation $\{5, 4\}$ by connecting midpoints of successive sides of the pentagons. Bomford carried out this construction on the plot of $\{5, 4\}$ that I had sent him. This technique works in general: the uniform tiling $(p.q.p.q)$ can be constructed from $\{p, q\}$ by connecting midpoints of successive sides of the regular p-sided polygons.

Figure 8 shows the tiling $(4.5.4.5)$ in bold lines superimposed on top of the underlying tessellation $\{5, 4\}$ shown in thin lines. The squares in rug 15 are each divided into four 36-36-90 degree isosceles triangles which are colored in five shades of tan and brown. The pentagons are each divided into ten 36-45-90 degree triangles which are colored in five shades of green. Bomford named rug 15 simply *Squares and Pentagons.* Rug 15 was started on March 20, 1985 and completed February 11, 1986. It is the same size as rugs 12 and 13.

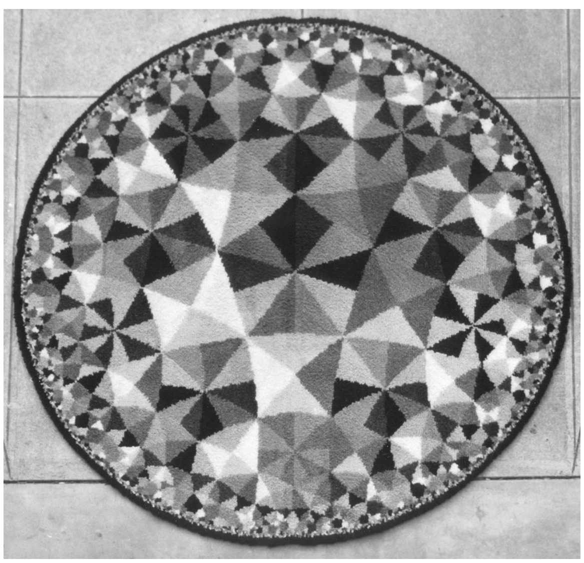

Figure 7: Bomford's rug 15 *Squares and Pentagons.*

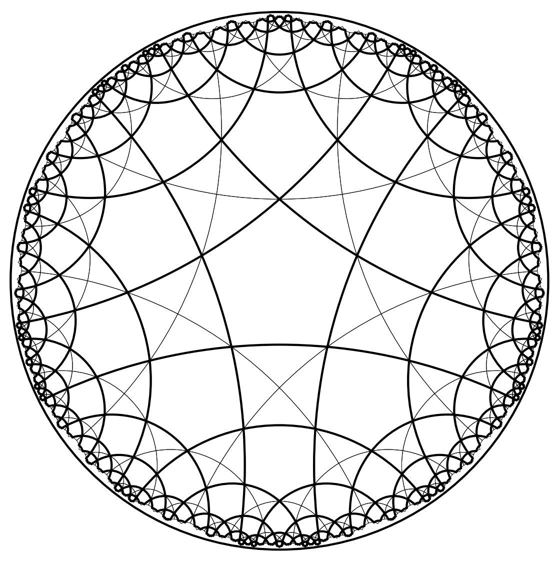

Figure 8: The tiling (4.5.4.5) (bold lines) superimposed on the tessellation $\{5, 4\}$ (thin lines).

Rug 18 is also based on the tiling $(4.5.4.5)$ and was designed to bring out the $(4.5.4.5)$ pattern more clearly. Rug 18 was not quite completed before Tony Bomford passed away.

5. Rugs 16 and 17

Figure 9 shows Bomford's rug 16, which he named *Triangles and Heptagons.* Much as rug 15 was inspired by my plot of the $\{5, 4\}$ tessellation, Bomford used my plot of the $\{7, 3\}$ tessellation to make the pattern for rug 16. Bomford added the uniform (3.7.3.7) tiling to the plot and sketched in the colors on it. Figure 10 shows the (3.7.3.7) tiling (bold lines) superimposed on the tessellation $\{7, 3\}$ (thin lines). Both the triangles

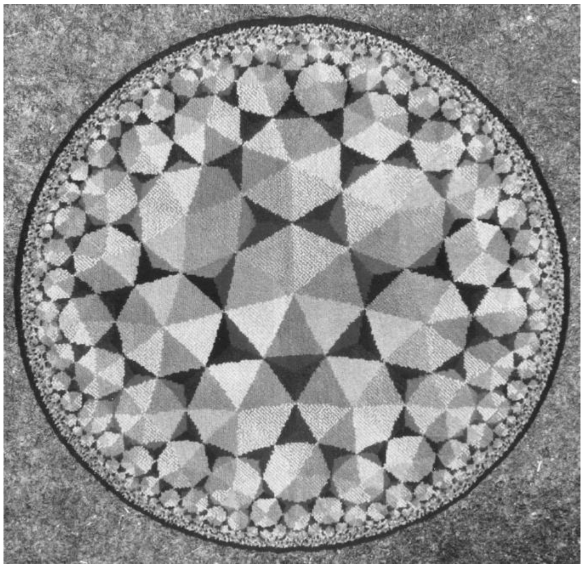

Figure 9: Bomford's rug 16 *Triangles and Heptagons.*

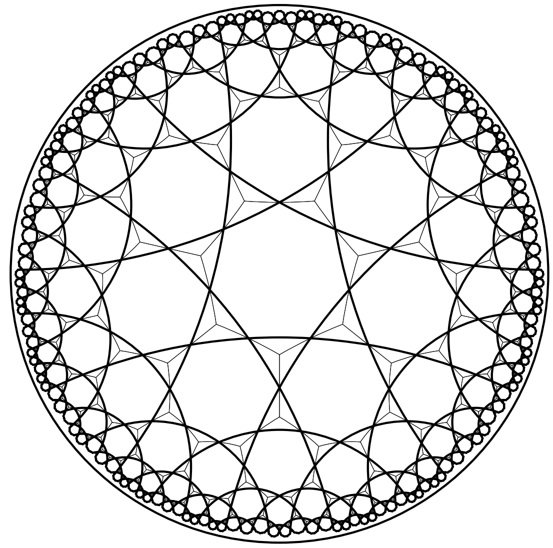

Figure 10: The tiling (3.7.3.7) (bold lines) superimposed on the tessellation $\{7, 3\}$ (thin lines).

and the heptagons in rug 16 are divided isosceles triangles. Eight shades of reddish colors from yellow to very dark red are used to color the isosceles subtriangles of the regular triangles. The isosceles subtriangles of the heptagons are colored by eight "tans", but only four colors of wool — four other tans are made by making four "tweeds" of the original four colors (by alternating two of the colors between neighboring knots). Each shade of red is associated with a shade of tan. Rug 16 was started on July 11, 1986 and finished July 25, 1987. It is the same size as all the other hyperbolic rugs.

Rug 17 was also based on the tiling (3.7.3.7). It was colored with greens and blues instead of reds to make the association between those colors and the tans more evident. Unfortunately it was stolen en route to an exhibition.

6. Conclusions and Future Work

We have examined the mathematical foundations of Tony Bomford's hyperbolic hooked rugs. Since all of his rugs had a mathematical theme, there are still twelve Euclidean rugs to analyze.

Also, if any hooked rug makers are interested in making new hyperbolic rugs, I would certainly be happy to work with them. To my knowledge no one has made a hyperbolic hooked rug with interlocking motifs such as are seen in Escher's *Circle Limit* patterns.

Acknowledgment

I would like to thank Richard Bomford for his generous help with the biographical details of his father's life.

References

[Coxeter1] H. S. M. Coxeter, Crystal symmetry and its generalizations, *Royal Society of Canada*, (3), 51 (1957), pp. 1–13.

[Coxeter2] H. S. M. Coxeter, Angels and Devils, in *The Mathematical Gardner,* David A. Klarner, Ed., Prindle, Weber and Schmidt, 1981, 197–209. (3), 51 (1957), pp. 1-13.

[DiFranza1] Happy and Steve DiFranza, *Rug Hooking,* Volume VIII, Number 3, November/December 1996. `http://www.rughookingonline.com/basic/howtohook.html`

[Grünbaum1] B. Grünbaum and G. C. Shepard, *Tilings and Patterns,* W. H. Freeman and Company, New York, 1986.

[Rumble1] Rumble, V., *Hooked Rugs* at:
`http://www.geocities.com/BourbonStreet/Quarter/2926/Rug_Making.html`