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Embroidery of a Hyperbolic Fish Pattern

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Outline

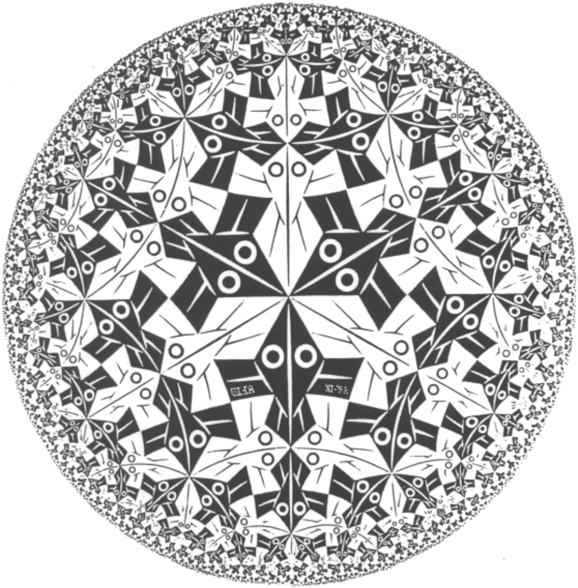
- Inspirations
- Background
- Some hyperbolic works in the fiber arts
- Conclusions and future work
- Contact information

Inspirations

Our first inspiration was M.C. Escher's four "Circle Limit" patterns, which motivated the first author, Dunham, to create a computer program that could replicate those patterns in 1980.

Secondly, last year Dunham visited a weaving and embroidery shop in Varanasi, India and wondered if embroidery techniques could also create "Circle Limit" patterns.

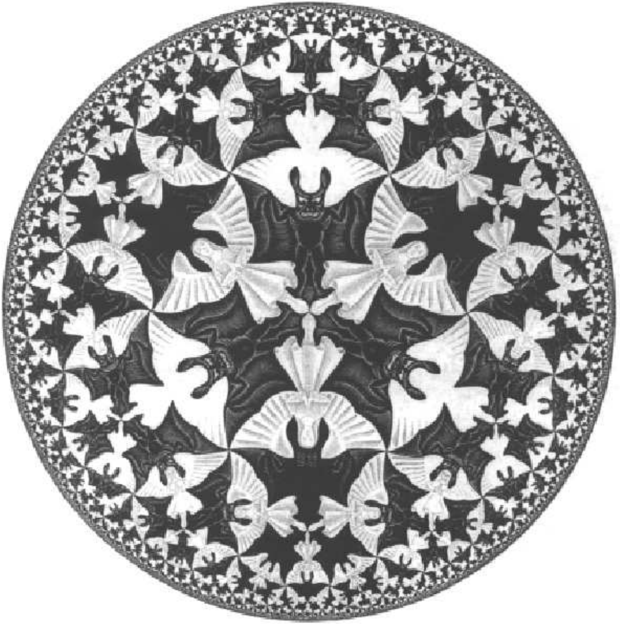
M.C. Escher's woodcut Circle Limit I



M.C. Escher's pattern Circle Limit II - my version

M.C. Escher's woodcut Circle Limit III

M.C. Escher's woodcut Circle Limit IV



An ordinary loom in Varanasi, India

A Jacquard loom in Varanasi, India

Background

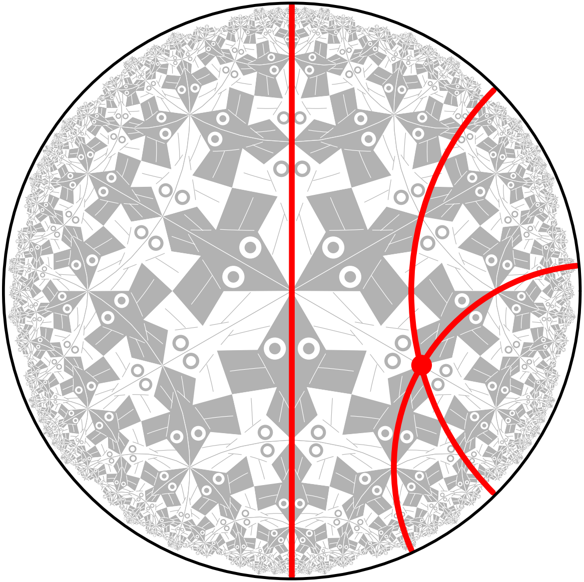
We review the following topics:

- Hyperbolic geometry
- The hyperbolic program can create other patterns in the pattern families of which Escher's *Circle Limits* are examples, in particular the pattern of the title slide.
- 3D hyperbolic fiber works.
- 2D hyperbolic patterns in the fiber arts.

Hyperbolic Geometry

- In 1901, David Hilbert proved that, unlike the sphere, there was no smooth isometric (distance-preserving) embedding of the hyperbolic plane into ordinary Euclidean 3-space.
- Thus we must use *models* of hyperbolic geometry in which Euclidean objects have hyperbolic meaning, and which must distort distance.
- One such model is the *Poincaré disk model*. The hyperbolic points in this model are represented by interior point of a Euclidean circle — the *bounding circle*. The hyperbolic lines are represented by (internal) circular arcs that are perpendicular to the bounding circle (with diameters as special cases).
- This model is appealing to artists since (1) angles have their Euclidean measure (i.e. it is conformal), so that motifs of a repeating pattern retain their approximate shape as they get smaller toward the edge of the bounding circle, and (2) it can display an entire pattern in a finite area.

Poincaré Disk Model of Hyperbolic Geometry



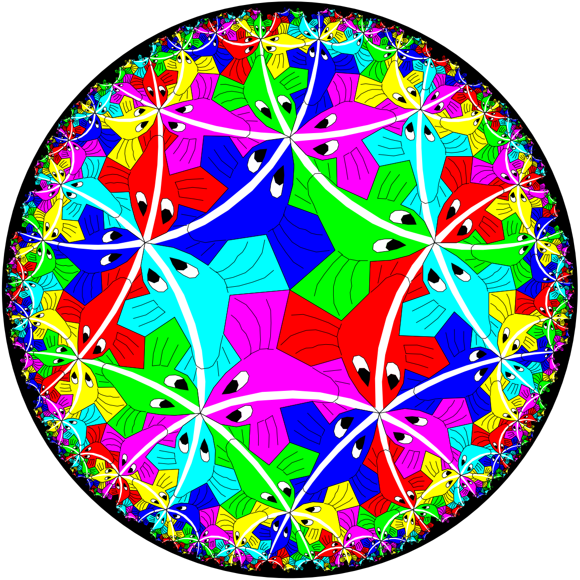
A Family of *Circle Limit III* Patterns

We use the symbolism **(p,q,r)** to denote a pattern of fish in which **p** meet at right fin tips, **q** meet at left fin tips, and **r** fish meet at their noses. Of course **p** and **q** must be at least three, and **r** must be odd so that the fish swim head-to-tail (as they do in *Circle Limit III*).

Escher's *Circle Limit III* pattern itself would be labeled (4,3,3) in this notation.

We note that our (5,3,3) pattern of the title slide requires six colors, whereas *Circle Limit III* only needs four. In fact the color symmetry group of our pattern is the alternating group $A(5)$ (not the symmetric group as in the abstract).

The (5,3,3) pattern of the title slide



Hyperbolic Geometry and the Fiber Arts

- 3D works
  - Diana Taimina's crochet works
  - Gabriele Meyer's reinforced crochet works
  - S. Louise Gould's triply repeating polyhedra
  - Barbara Nimershiem's Borromean rings quilts
- 2D patterns using the Poincaré model
  - Tony Bomford's hooked rugs
  - Mary Williams' quilt
  - Our embroidered fish pattern using an automated embroidery machine and software

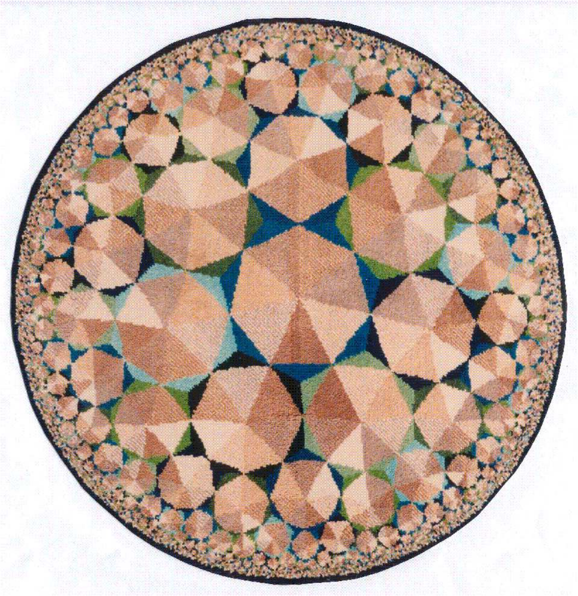
Diana Taimina's $\{6,4\}$ surface

Gabriele Meyer's "Red Rose"

Louise and Frank Gould's (5.5.5.5.5) surface

Barbara Nimershiem's Borromean Rings Quilt II (closed)

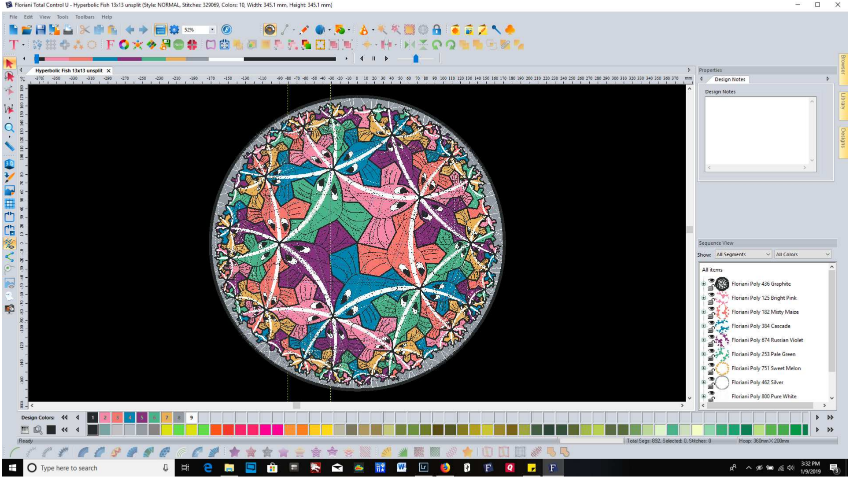
Tony Bomford's Rug 17



Mary Williams' quilt "Poincaré"

Husqvarna Epic embroidery machine

Floriani Total Control embroidery software



Our embroidered "(5,3) Fish Pattern"

Future Work

- There are many more hyperbolic circle patterns that could be embroidered, some more easily than others.
- And, inspired by fiber artists who have worked in 3D, we would like to embroider Escher-like patterns on 3D surfaces.
- We would also like to explore papercrafting a hyperbolic pattern with a digital cutter/printer.

Acknowledgements and Contact

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