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Using Papercrafting to Create Hyperbolic Patterns

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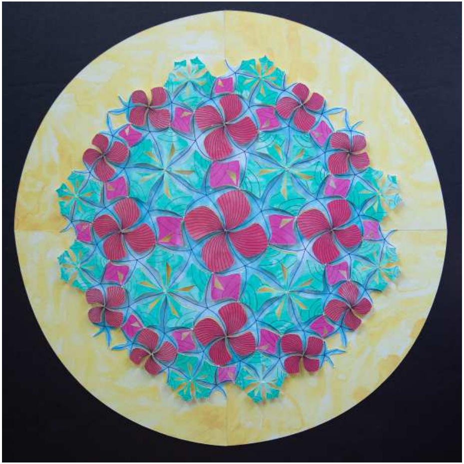


Outline

- Inspiration
- Background — Theory
- The papercrafted hyperbolic shell pattern
- A free-standing lace dodecahedron
- Conclusions and future work
- Contact information

Inspiration

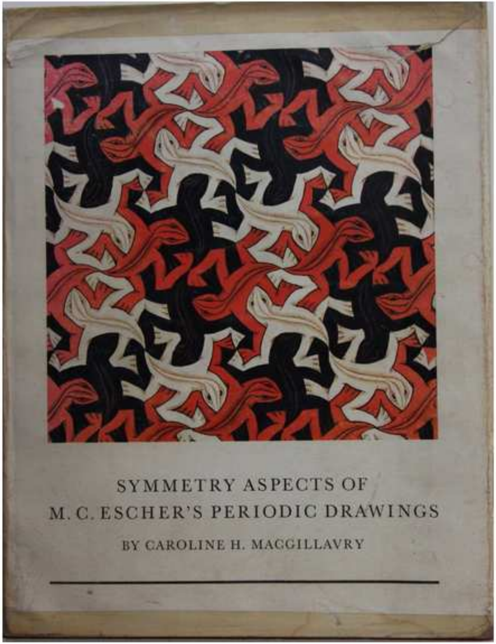
Our basic inspiration was M.C. Escher's Regular Division Drawing 42, a pattern of shells.

We first encountered this pattern in Carolina MacGillavry's book *Symmetry Aspects of M.C. Escher's Periodic Drawings* (1965).

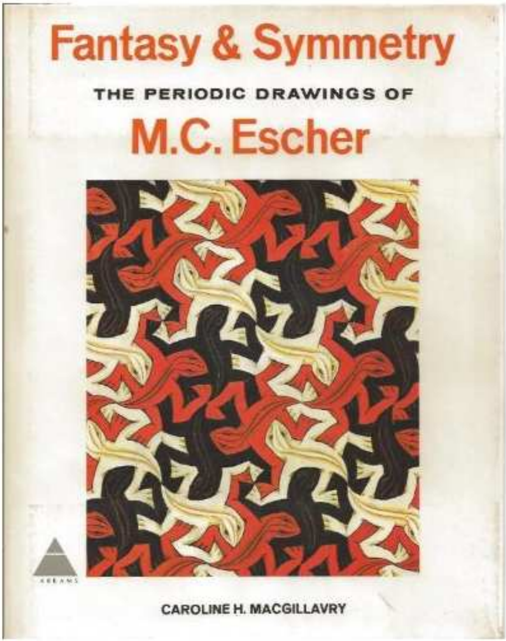
This book was later reprinted as *Fantasy and Symmetry: The Periodic Drawings of M.C. Escher* (1976).

The definitive book on Escher's periodic patterns is Doris Schattschneider's book *M.C. Escher: Visions of Symmetry* (1990, 2004).

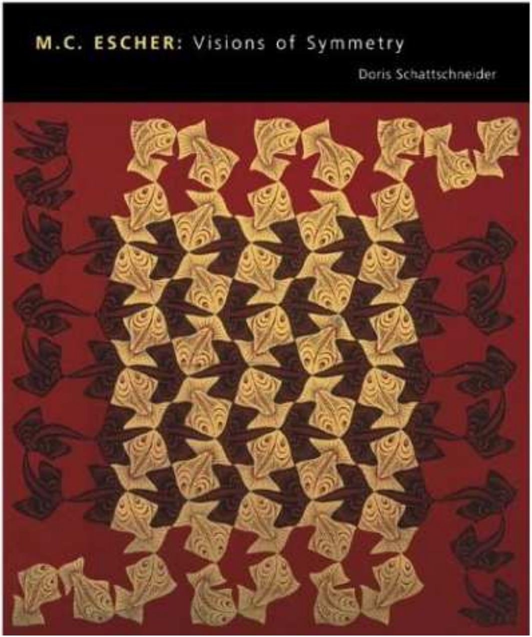
Symmetry Aspects of M.C. Escher's Periodic Drawings



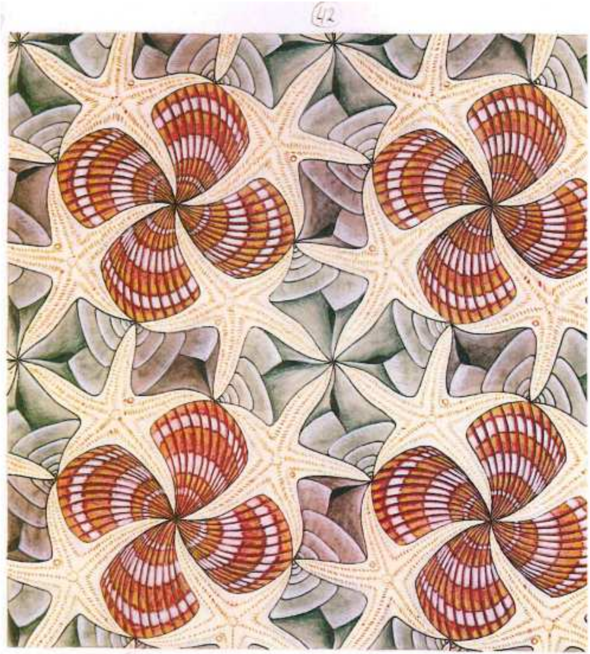
Fantasy and Symmetry: The Periodic Drawings of M.C. Escher



M.C. Escher: Visions of Symmetry



Regular Division Drawing 42

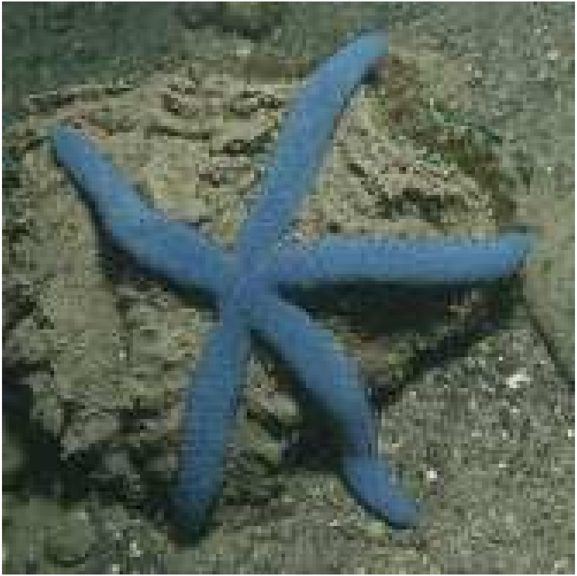


The shells in Regular Division Drawing 42

- Note: I don't think these are exact representations of sea animals. Here are my interpretations (for convenience of discussion).
- The red shells are scallops.
- The light green shells are conchs.
- The gray shells are snail shells.
- The 'background' starfish are light tan / yellow.

Actual Sunset Scallops

Micronesian Blue Sea Star —
color used for some stars in papercrafted work



Background — Theory

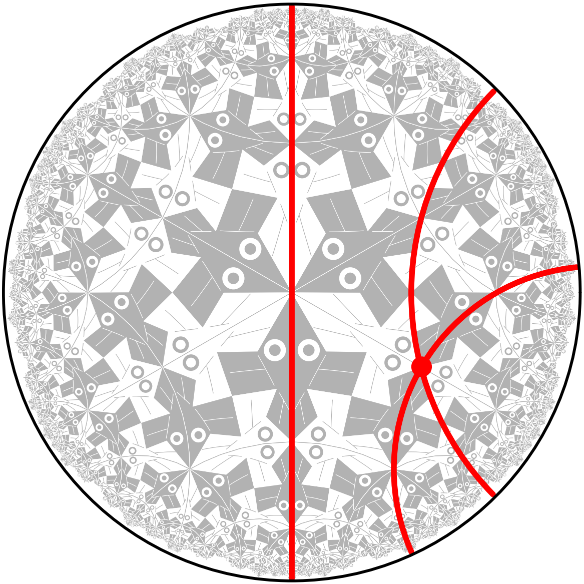
We discuss the following topics:

- The symmetry of Regular Division Drawing 42 is $p4$ or 442 in orbifold notation.
- Hyperbolic geometry
- Our hyperbolic program can create patterns that come in "families" that are parameterized by combinatorial indices — the orders of rotations, for example.
- One can imagine shell patterns with symmetry group $pq2$ (orbifold notation) where p and q are the orders of the rotations about the conchs. If $(p-2)(q-2)$ is less than, equal to, or greater than 4, the pattern will be spherical, Euclidean, or hyperbolic respectively.

Hyperbolic Geometry

- In 1901, David Hilbert proved that, unlike the sphere, there was no smooth isometric (distance-preserving) embedding of the hyperbolic plane into ordinary Euclidean 3-space.
- Thus we must use *models* of hyperbolic geometry in which Euclidean objects have hyperbolic meaning, and which must distort distance.
- One such model is the *Poincaré disk model*. The hyperbolic points in this model are represented by interior point of a Euclidean circle — the *bounding circle*. The hyperbolic lines are represented by (internal) circular arcs that are perpendicular to the bounding circle (with diameters as special cases).
- This model is appealing to artists since (1) angles have their Euclidean measure (i.e. it is conformal), so that motifs of a repeating pattern retain their approximate shape as they get smaller toward the edge of the bounding circle, and (2) it can display an entire pattern in a finite area.

Poincaré Disk Model of Hyperbolic Geometry



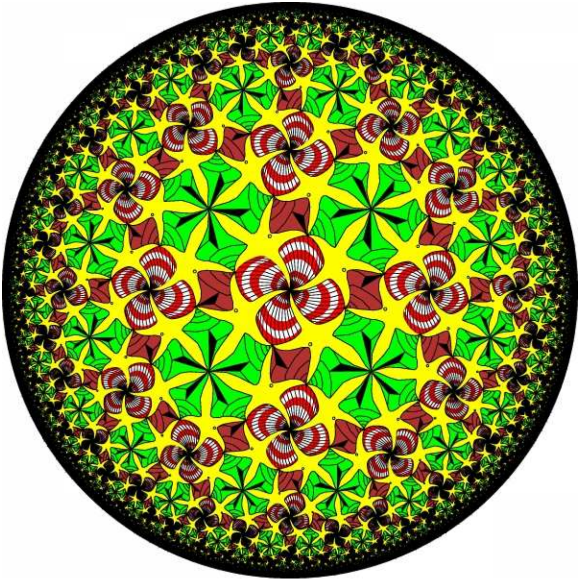
A Family of Shell Patterns

We use the symbolism **(p,q)** to denote a pattern of shells with symmetry group $pq2$ in orbifold notation.

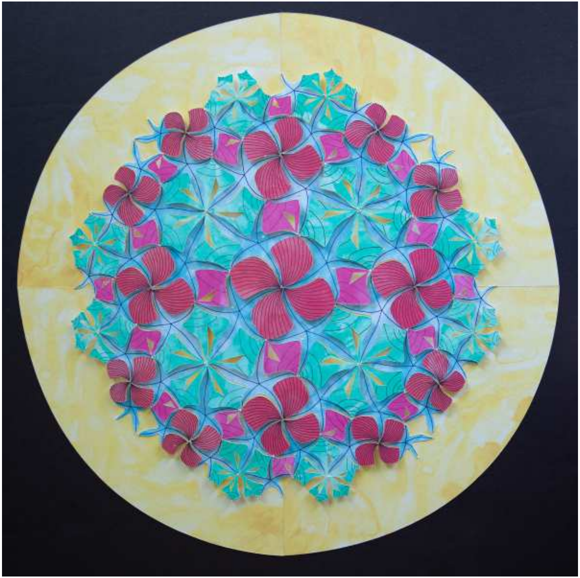
There are two kinds of rotation centers at the meeting points of conch shells — with orders p and q.

Thus Escher's Regular Division Drawing 42 would be labeled (4,4) in this notation.

The (5,5) pattern used as a model for the title slide



The papercrafted version of (5,5) shown in the title slide



Artistic considerations

- Color — we tried to choose colors found in the sea.
- Paper — there are many kinds of colored paper to choose from, with and without patterns.
- A partial 3D effect.

Color

The colors chosen were as follows:

- The scallops: sunset red
- The conchs: light blue
- The snails: pink
- The starfish: sea star blue (inside) or yellow (outside)

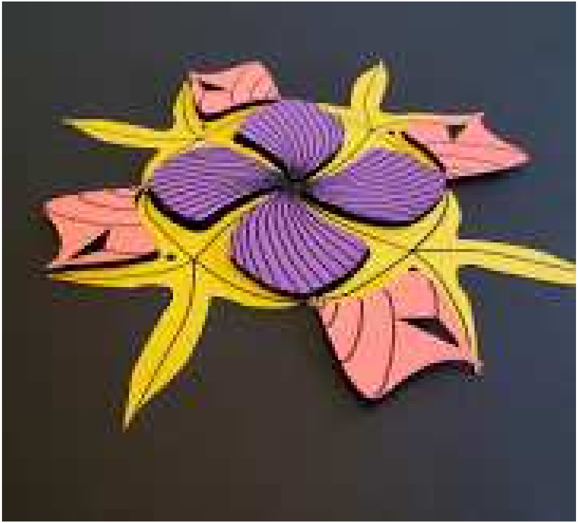
Paper choice

- When using patterned paper, the pattern remains at the same scale, whereas the shells get smaller toward the bounding circle.
- Using solid color paper solves this problem, but looks boring.
- Cure: use watercolor paper, which has features at all scales.

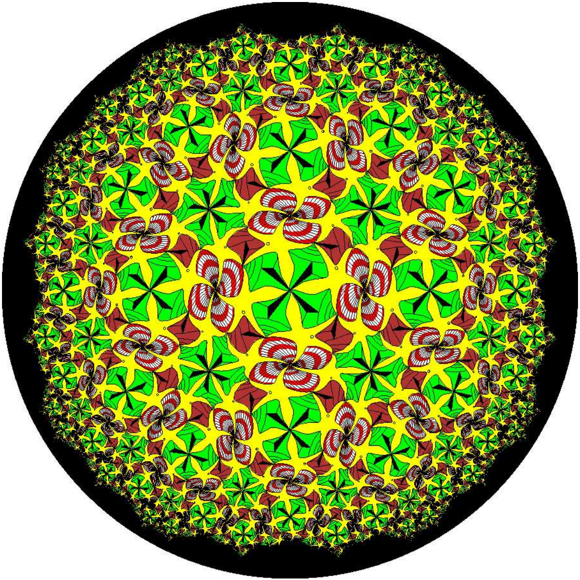
The slight 3D effect

- The scallops, conchs, and snails were raised slightly by putting double sticky foam tape under them. The starfish are on the surface. They touch and divide the other shells.
- The result gives the work a slight 3D effect.
- This allows for interesting plays of light and shadow not found in the original paper drawings.

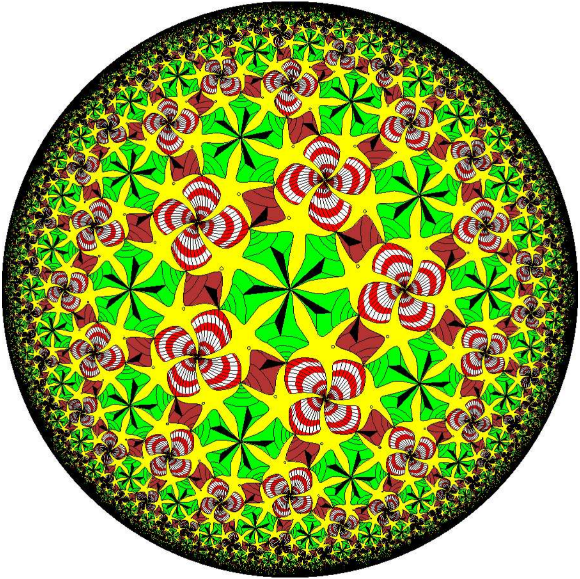
An oblique view that shows the 3D effect



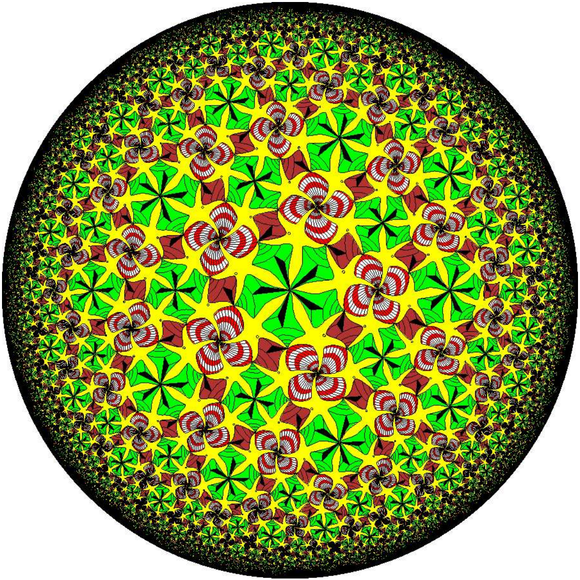
A (5,4) pattern



The (5,5) pattern — conch centered



The (5,4) pattern — conch centered



A Free Standing Lace Dodecahedron

We were interested in making polyhedra out of free standing lace.

The eventual goal was to make a free standing lace dodecahedron with each face decorated by a letter from the Greek alphabet that had mathematical significance.

Also the color of the letters would be evenly spaced around the color wheel and the background would be offset by one color from the letter. So a total of 6 colors were used: red, orange, yellow, green, blue and purple.

But to get experience with free standing lace polyhedra, Lisa decided to make a cube first. There was a 'flatness' problem along the way, but it was resolved. On the other hand the dodecahedron presented more of a challenge.

A finished square on stabilizer

A square with the stabilizer dissolved

Ironed squares

The finished cube

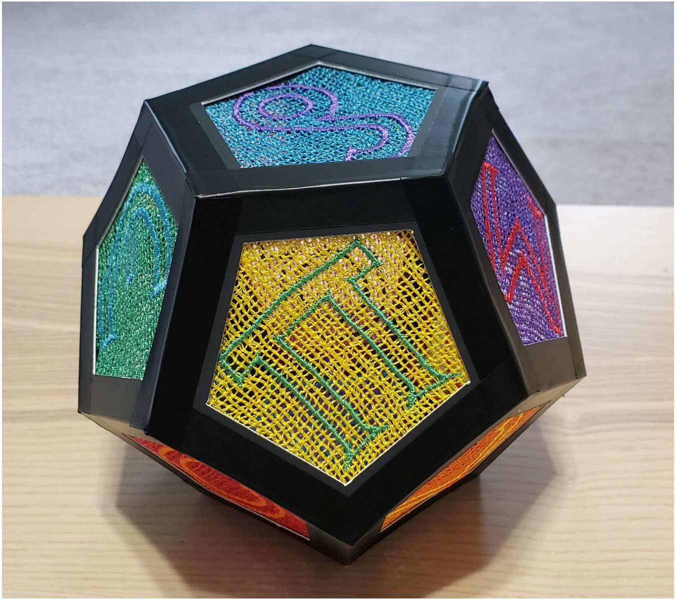
A pentagon with Π

A better pentagon with Π

An arrangement of the 12 pentagons

The free standing lace dodecahedron

The free standing lace dodecahedron with bracing — provided by a papercrafted framework of posterboard and sticky back vinyl



Conclusions and Future Work

- Both papercraft and machine embroidery are done with craft-scale computer aided manufacturing, just as with 3D printing.
- They are entry points to learning about advanced manufacturing and robotics, both subjects of intense interest to students, parents, local governments, and potential new employers
- This type of work is of interest to students who are more tactile learners than visual learners, a group that traditional education sometimes has trouble reaching.
- The two media chosen here also tend to appeal to females, a group is often underrepresented in higher mathematics.
- There are many more hyperbolic circle patterns that could be papercrafted, some more easily than others.
- We would like to explore creating patterned triply repeating polyhedra using papercrafting or free standing lace.

Acknowledgements and Contact

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