**Lesson Topic:** Probability  
**Grade level:** 6th Grade  
**Length of lesson:** 50 minutes  
**Teacher:** Eric Anderson

### Stage 1 – Desired Results

**Content Standard(s):**  
**Standard:** Use probabilities to solve real-world and mathematical problems; represent probabilities using fractions, decimals and percents.

**Benchmark 6.4.1.1:** Determine the sample space (set of possible outcomes) for a given experiment and determine which members of the sample space are related to certain events. Sample space may be determined by use of tree diagrams, tables, or pictorial representations.

**Understanding (s)/goals:**  
- Students will understand that by using the counting principle, we can determine the number of possible outcomes for a probability experiment.  
- Students will understand that by determining the set of possible outcomes (sample space) for a given experiment, we can predict how likely a specific outcome is to occur.

**Essential Question(s):**  
- How does the counting principle help us to determine possible outcomes?  
- How does knowing the sample space help us predict the likelihood that an event will occur?  
- How can we use the counting principle to help us make real-life decisions?

(See Note 1)

### Student objectives (outcomes):  
Students will be able to:

**All students will be able to:**  
- Record outcomes of flipping a coin multiple times  
- Use a tree diagram to determine possible outcomes of an experiment  
- Use a table to determine possible outcomes of an experiment  
- Use the counting principle to find the total number of possible outcomes

**Most students will be able to:**  
- Explain why all of the possible outcomes are not equally likely

**Some students will be able to:**  
- Explain why a game or an experiment is fair or unfair

(See Note 2)
**Stage 2 – Assessment Evidence**

**Performance Task(s)**
- Conduct a probability experiment using coins, dice, or spinners and record the results OR
- Create a small poster showing how probability concepts play a part in our everyday lives OR
- Create a song or poem with lyrics that explain the probability concepts discussed in the lesson

**Other Evidence:**
- Completion of examples during class
- Discussion with peers
- Counting Principle worksheet (homework)

*(See Note 3)*

**Stage 3 – Learning Plan**

**Learning Activities:**

**Materials:**
- Coins
- Candy
- Spinners
- Dice
- Smart Board examples
- Counting Principle worksheet

**Warm Up: 5 minutes**
Have students stand up by their chairs. Using a spinner on the Smart Board, have each student select one of the four colors on the spinner. Spin the spinner and if it lands on the color a student chose, that student sits down. Repeat until only one color is left to be spun. Students still standing “win.” Repeat, but this time have students do jumping jacks (or other basic physical activity). If their color is spun, they stop doing the physical activity. Those left doing the physical activity after three different colors have been spun, “win.” *(See Note 4)*

**Intro: 5 minutes**
Ask students if they want a piece of candy (all hands should go up!). Have two volunteers come to the front of the room. Ask the class what would be a fair way to decide who gets the piece of candy. A typical answer should be by flipping a coin, but there might be other answers as well (rock-paper-scissors, class vote, etc.). Ask students which method we should use. Have them defend their answers. Ask if all methods mentioned are fair. Ask the two volunteers which method they would prefer. State that we will be deciding by flipping a coin, but before flipping it, ask the class why flipping a coin is fair. Desired answers would include: because they both have an equal chance of winning, they both have a 50% chance of winning, etc. Some students might think flipping a coin is unfair. Have them explain why they think so. Let one volunteer choose heads or tails and the other volunteer will be the side not chosen by the first volunteer. Flip the coin and award the piece of candy to the winner. Explain to the class that we have just done a probability experiment. To confirm their understanding, have
students determine what the probability of student A winning the piece of candy was (1/2 or 50%) and what the probability of student B winning the piece of candy was (1/2 or 50%). (See Note 5)

**Activity: 10 minutes**

Now ask students what they think about tossing 3 coins. What are the possible results? Make sure students understand that responding with a wrong answer is no big deal. (See Note 6) Do the 3 coin flip experiment with the class. The teacher wins when 2 coins are the same and 1 is different (i.e. 2 heads and 1 tails). The students win when all 3 coins are the same (i.e. all 3 coins are heads). Choose 3 volunteers to flip the coins 20 times and have the rest of the students keep track of the combinations. Ask students if it is a fair game (both sides have an equal chance of winning). If they say yes or no, tell them to explain why.

**Teacher Note:** There are 8 combinations possible when flipping 3 coins: (H,H,H), (H,T,H), (H,T,T), (H,H,T), (T,H,T), (T,T,H), (T,H,H), (T,T,T). The teacher has 6 of the 8 combinations (75%) while the students have 2 of the 8 combinations (25%). Most students will not be able to explain why they think it is not a fair game.

**Lesson: 20 minutes**

Before moving into the lesson, ask students how things would change if 4 coins were flipped. What about 5 coins? Students should realize that there would be more possible outcomes and the game would still not be fair (actually, less fair than with 3 coins). (See Note 7) Introduce the key term sample space to the students (have them write it down!). Sample space is a number of people, objects, or events chosen from a given population to represent the entire group (i.e. heads or tails in the intro experiment). Ask the students if anyone has ever heard of a tree diagram (a few hands may go up). A tree diagram is a pictorial way of representing combinations of things. Use tree diagrams to show the number of possibilities (combinations) for a few example problems. See attached tree diagram examples (examples will be shown using Smart Board). Make sure students understand that each “branch” of the tree diagram represents a possible outcome of the probability experiment. Introduce the key terms combinations, outcome, and event. Have students go back to the 3 coin flip experiment and show all the combinations for flipping 3 coins using a tree diagram. Show the correct tree diagram for the 3 coin flip experiment. Discuss why the teacher had a better probability of winning: How many combinations were there where the teacher won? (6) How many combinations were there where the students won? (2) Show why the teacher had a better chance of winning using a spinner (75% of spinner = “teacher wins”, 25% of spinner = “students win”).

QUICK BREAK (2 minutes): Have students get out of their chairs and move around a bit. Nothing too extravagant, just a short break from sitting so students can stretch and mentally regroup before the rest of the lesson. (See Note 8)

Ask students how we would determine how many possible outcomes there are for a
probability experiment without using a tree diagram? Show one more tree diagram example (attached – example 4) and emphasize the fact that there were THREE shirt color options and TWO shoe color options and we ended up with 6 possible outcomes. Students should discover that by multiplying the shirt options by the shoe options results in the total possible outcomes. Introduce the key term counting principle. The counting principle is where you multiply the number of possible outcomes of one event (i.e. shirt colors) by the number of outcomes from other events (i.e. shoe colors) to find the total number of possible outcomes. Show several examples using the counting principle. Ask students if they can think of any situations in their lives where the counting principle could be used.

**Closing Activity/Homework: 10 minutes**

Ask students if they can think of any other ways to represent/show all the possible outcomes of a probability experiment. Show an example of using a table to list all the possible outcomes of a probability experiment. Students now have three options. One is to use coins, dice, or spinners to conduct a probability experiment. Each student will flip a coin, roll two dice, or spin a spinner 20 times and record their results. The next option is to create a small poster showing how probability concepts play a part in our everyday lives. The third option is to create a song or poem with lyrics that explain the probability concepts discussed in the lesson. *(See Note 9)*

Review the main concepts that were addressed today (tree diagrams, counting principle, tables, etc.) and how they are related to each other. Assign Counting Principle worksheet. With any remaining class time, students can work on the worksheet and ask any questions they might have. *(See Note 10)*
Note 1: A common question students have during a math class is, "Why do we have to know this?" Sometimes teachers give the simple answer by saying students need to know it for the test or to succeed in upcoming math classes. Ideally, though, students should be able to see how any given math concept applies to their every day lives or their future lives. Educators need to design educational experiences that encourage a relevant emotional connection to the material being learned (Immordino-Yang and Faeth, 2010). Emotionally competent stimuli (ECS) are key for effective learning (Zull, 2011). If students can become emotionally attached to a concept, they will be more excited to learn about it and retain the information for a longer period of time. The goal of this lesson is to get students excited about probability and for them to understand how they can use probability concepts in their everyday lives. Brain systems are shaped by experience (Begley, 2008). Students who experience hands on probability experiments and models will be better prepared to use their probability knowledge in future situations.

Note 2: Every brain is wired differently (Medina, 2008). Some students will pick up on the probability concepts right away, while others might struggle with them. This doesn’t mean the students who are struggling aren’t intelligent; they just aren’t able to wrap their minds around the concepts as quickly as others. As educators, we need to balance confidence and challenge (Zull, 2011). If the material is too hard for all students, they will get frustrated. On the other hand, if the material is too easy, students will get bored. It works well to separate student objectives to make it clear for yourself what you expect all students to be able to do by the end of the lesson, and things you expect most and some students will be able to do. This way, the more advanced students will still be challenged by at least some aspect of the lesson, and the less advanced students will be confident in their ability to do at least some parts of the lesson. For the students that seem to be struggling and frustrated, however, teachers need to remember that what looks like resistance is often a lack of clarity (Heath and Heath, 2010). Teachers need to be sure to be very clear when teaching a concept and when giving directions.

Note 3: Emotion is most effective at facilitating the development of knowledge when it is relevant to the task at hand (Immordino-Yang and Faeth, 2010). Students are given options for a performance task in the hopes that one strikes their emotion and excites them. The options play to different intelligence strengths students may possess. The more of an individual's intelligences you can appeal to when making an argument, the more likely you are to change a person’s mind (Gardner, 2004). At first, some students might not be interested in probability. The goal, though, by giving options for assignments, is that students will see a way they can express their interests. One student might love music or poetry and choose to create a song or poem relating to probability. Another student might enjoy art and drawing and choose to create a poster about probability. By appealing to these different intelligences of students, a teacher can effectively “change a person’s mind” about a concept, in this case, probability. It’s important for students to feel some emotion for concepts learned in school. People with well-developed emotional skills are more likely to be content and effective in their lives.
(Goleman, 2006). Children spend most of their childhood in school so what better place to develop emotional skills than in the classroom.

**Note 4:** Physical activity sparks biological changes that encourage brain cells to bind to one another (Ratey, 2008). Doing some sort of physical activity at the beginning of a class period will help students learn more effectively. Exercise boosts brainpower and zaps harmful stress chemicals (Medina, 2008). Students have many things that go on in their lives outside of school that may be causing them stress or frustration. The goal of some physical activity, even if it's as little as standing up and sitting down or doing jumping jacks, is to free students of outside stresses or pressures they may be feeling and help them focus on the upcoming lesson.

**Note 5:** Learning takes place through action, but is driven by emotion (Zull, 2011). What student isn’t going to be emotionally invested in a lesson that starts with the opportunity for candy? Students will get excited to learn about probability. Factual knowledge is useless without guiding emotional intuition (Immordino-Yang and Faeth, 2010). A teacher can state fact after fact to his or her students, but the majority of them won’t truly care unless they have some sort of emotional attachment to the content. There is a link between attention and learning – we don’t pay attention to boring things (Medina, 2008). Students are much more likely to pay attention during a lesson if they are interested in what is being taught or how it is being taught. For this reason, it’s crucial to create learning experiences that help students see the importance of a concept (Zull, 2002).

**Note 6:** A “mistake-rich” environment is preferable. It produces a better education and leads to more insight and truth (Zull, 2011). Students need to feel comfortable answering questions, even if they aren’t sure they are correct. The school’s social and cultural climates affect learning (Sousa, 2010). If a student answers a question incorrectly and the rest of the class starts laughing, what are the chances that student attempts to answer another question? The chances are slim. It is important for a teacher to create an atmosphere of trust and respect in his or her classroom. Students will allow themselves to experience failure (Immordino-Yang and Faeth, 2010).

**Note 7:** Learning can build on pre-existing brain networks to achieve new functions (Posner, 2010). First of all, students may have some prior knowledge about probability before beginning this lesson, either from a previous grade or just general knowledge. Also, at this point in the lesson, students should be able to use their “prior knowledge” from earlier in the lesson to infer possible results if four or five coins were to be flipped. It is important to relate new information to prior knowledge (Willis, 2010). Students will be able to better understand new information that is being presented if they can make a connection between the new information and something they already know. The most connections in the brain are made between the ages 0-3 and during puberty (Medina, 2008). So for students, especially those in middle school, their brains are firing at unbelievable speeds, taking in new information and making connections between things they are learning and things they already have learned.
**Note 8:** Physical activity has a positive influence on memory, concentration, and classroom behavior (Ratey, 2008). Having students take a short break midway through a lesson can be very helpful in different ways. It gives students a chance to quickly unwind and relax so they are able to better concentrate on the rest of the lesson. There are multiple motivations of behavior (Maslow, 1943). Whether a student is a chronic misbehaver or is just having a bad day, he or she is less likely to act out or be disruptive after having a break. Also, movement enhances learning and memory (Sousa, 2010). There is a better chance students remember what they have learned and what they are about to learn when they have a chance to take part in physical activity, even if it’s minimal. Exercise improves problem solving, planning, and attention (Medina, 2008). Obviously, there is a lot of problem solving in a math class, so the importance of physical activity is clear here. Also, exercise improves students’ attention skills. This is important especially for students who struggle to pay attention regularly, such as a student with ADHD. Teachers should try to work in at least a little physical activity into their lessons. The benefits far exceed the downfall of having to take some time out of actually teaching for physical activity.

**Note 9:** Simply put, lectures don’t work (Don’t Lecture Me, 2011). A lot of the time for math, some lecture will be needed. That’s just the nature of the subject. But as you can see from this lesson, there are many ways to stray away from lecture when teaching math. It is important to break up lectures to avoid habituation (Zull, 2002). Breaking up a lecture can mean doing an activity or having a discussion or even just taking a break for a couple minutes, as I’ve included in the lesson above. It is impossible for students to take in all the information of a lecture (Don’t Lecture Me, 2011). The vast majority of students do not enjoy sitting through a class period where the teacher just lectures the whole time. Students need some variety.

**Note 10:** One of the most important things for a teacher to do is see things as students see them (Zull, 2002). A lot of teachers are so knowledgeable about their given subject, they assume that students should easily pick up on the concepts. Obviously, this is rarely the case. Teachers should try to understand where students are coming from. The Theory of Mind is the ability to understand interior motivations of someone and construct a predictable “theory of how their mind works” (Medina, 2008). The most successful teacher understands how each of his or her students’ minds work and uses relative teaching strategies. Teachers should relate the material to the lives and interests of students (Immordino-Yang and Faeth, 2010). This way, students will be able to see the material in a way that interests them and be better able to understand the concepts.
Smart Board Material

**Probability** – the chance of an event occurring. For example, the chance the coin flip resulted in heads was \( \frac{1}{2} \).

ACTIVITY – 3 coin flip experiment

Teacher wins: 2 of the 3 coins are the same
Students win: all 3 coins are the same (all heads or all tails)

Is this a fair game? Why or why not?

Teacher wins: \((H,H,T), (H,T,T), (H,T,H), (T,H,T), (T,H,H), (T,T,H)\) = 6

Students win: \((H,H,H), (T,T,T)\) = 2

NOT FAIR!!

**Sample Space** – Sample space is a number of people, objects, or events chosen from a given population to represent the entire group (i.e. heads or tails in the intro experiment).

What is the sample space for the 3 coin flip experiment?

**Tree Diagram** – a pictorial way of representing combinations of things.

Probability – Tree Diagrams

Example 1:

<table>
<thead>
<tr>
<th>Meat</th>
<th>Cheese</th>
<th>* Draw Tree Diagram Here *</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ham</td>
<td>Cheddar</td>
<td>(6 outcomes)</td>
</tr>
<tr>
<td>Turkey</td>
<td>Pepper jack</td>
<td></td>
</tr>
<tr>
<td>Pork</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Combinations** – the different ways a group of items or events can be chosen.

**Outcome** – one of the possible events in a probability situation.

**Event** – a possible outcome in probability.

Example 2:

<table>
<thead>
<tr>
<th>Snack</th>
<th>Drink</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nachos</td>
<td>Pop</td>
</tr>
<tr>
<td>Pretzel</td>
<td>Water</td>
</tr>
<tr>
<td>Pizza Slice</td>
<td>Gatorade</td>
</tr>
</tbody>
</table>

* Draw Tree Diagram Here *

(9 outcomes)

Example 3:

<table>
<thead>
<tr>
<th>Ice Cream</th>
<th>Cone</th>
<th>Toppings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chocolate</td>
<td>Regular</td>
<td>Sprinkles</td>
</tr>
<tr>
<td>Vanilla</td>
<td>Waffle</td>
<td>Hot Fudge</td>
</tr>
</tbody>
</table>

* Draw Tree Diagram Here *

(8 outcomes)

Create a tree diagram for the 3 coin flip experiment
Three Coin Flip Experiment:

<table>
<thead>
<tr>
<th>Coin 1</th>
<th>Coin 2</th>
<th>Coin 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heads</td>
<td>Heads</td>
<td>Heads</td>
</tr>
<tr>
<td>Tails</td>
<td>Tails</td>
<td>Tails</td>
</tr>
</tbody>
</table>

* Draw Tree Diagram Here *

(8 outcomes)

How can we determine the number of possible outcomes without using a tree diagram?

Example 4:

<table>
<thead>
<tr>
<th>Shirt</th>
<th>Shoes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue</td>
<td>Black</td>
</tr>
<tr>
<td>Red</td>
<td>Brown</td>
</tr>
<tr>
<td>Green</td>
<td></td>
</tr>
</tbody>
</table>

* Draw Tree Diagram Here *

(6 outcomes)

**Counting Principle** – multiplying the number of possible outcomes of one event by the number of outcomes from other events to find the total number of possible outcomes.

Counting Principle Examples

Example 1:

How many total choices?
4 baseball bats, 3 colors

Example 2:

How many total choices?
4 pizzas, 5 toppings
Example 3:

Charlotte bought a new jewelry collection with 4 sets of earrings and 6 necklaces. How many combinations of jewelry can she wear?

Representing Possible Outcomes Using a Table

Example:

<table>
<thead>
<tr>
<th>Fruit</th>
<th>Vegetable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apples</td>
<td>Carrots</td>
</tr>
<tr>
<td>Oranges</td>
<td>Beans</td>
</tr>
<tr>
<td>Bananas</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Carrots</th>
<th>Beans</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apples</td>
<td>(Apples, Carrots)</td>
<td>(Apples, Beans)</td>
</tr>
<tr>
<td>Oranges</td>
<td>(Oranges, Carrots)</td>
<td>(Oranges, Beans)</td>
</tr>
<tr>
<td>Bananas</td>
<td>(Bananas, Carrots)</td>
<td>(Bananas, Beans)</td>
</tr>
</tbody>
</table>

Create a table that represents all the possible outcomes when rolling two dice.

Rolling Two Dice

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1,1)</td>
<td>(1,2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* SAVE YOUR TABLE*

HOMEWORK – Counting Principle Worksheet
Sources Used:
(Ratey, 2008)
(Medina, 2010)
(Zull, 2011)
(Immordino-Yang and Faeth, 2010)
(Zull, 2002)
(Goleman, 2006)
(Sousa, 2010)
(Posner, 2010)
(Willis, 2010)
(Begley, 2008)
(Gardner, 2004)
(Heath and Heath, 2010)
(Maslow, 1943)
(Don’t Lecture Me, 2011)