Review for Test 3

Section 2.5 look up answers in back of book

1. Use \( f(x) = x^2 \) to obtain the graph of \( g(x) = (x - 2)^2 + 1 \) #59
2. Use \( f(x) = x^2 \) to obtain the graph of \( g(x) = -2(x + 1)^2 + 1 \) #65
3. Use \( f(x) = \sqrt{x} \) to obtain the graph of \( g(x) = \sqrt{-x} + 2 \) #73
4. Use \( f(x) = \sqrt{x} \) to obtain the graph of \( g(x) = -\sqrt{x} + 2 \) #71
5. Use \( f(x) = |x| \) to obtain the graph of \( g(x) = |x| + 4 \) #81
6. Use \( f(x) = |x| \) to obtain the graph of \( g(x) = |x + 4| - 2 \) #85
7. Use \( f(x) = \sqrt[3]{x} \) to obtain the graph of \( g(x) = \sqrt[3]{x} + 2 \) #107
8. ***Use \( f(x) = x^3 \) to obtain the graph of \( g(x) = \frac{1}{2}(x - 3)^3 - 2 \) #105

Section 2.6

Find \( f + g \), \( f - g \), \( fg \), \( f/g \) for each of the following pairs of functions and the domain for each.

1. \( f(x) = x - 6 \) and \( g(x) = 5x^2 \)
   \( f + g = 5x^2 + x - 6 \) All real numbers
   \( f - g = -5x^2 + x - 6 \) All real numbers
   \( fg = 5x^3 - 30x^2 \) All real numbers
   \( \frac{f}{g} = \frac{x - 6}{5x^2} \) \( x \neq 0 \) or \( (-\infty, 0) \cup (0, \infty) \)

2. \( f(x) = 2 + \frac{1}{x} \) and \( g(x) = \frac{1}{x} \)
   \( f + g = \frac{2x + 2}{x} \) \( x \neq 0 \) or \( (-\infty, 0) \cup (0, \infty) \)
   \( f - g = 2x \neq 0 \) or \( (-\infty, 0) \cup (0, \infty) \)
   \( fg = \frac{2x + 1}{x^2} \) \( x \neq 0 \) or \( (-\infty, 0) \cup (0, \infty) \)
   \( \frac{f}{g} = 2x + 1 \neq 0 \) or \( (-\infty, 0) \cup (0, \infty) \)
3. \( f(x) = \frac{3x+1}{x^2-25} \) and \( g(x) = \frac{2x-4}{x^2-25} \)

\[
\begin{align*}
f + g &= \frac{5x-3}{x^2-25} \quad x \neq \pm 5 \text{ or } (-\infty, -5) \cup (-5, 5) \cup (5, \infty) \\
f - g &= \frac{1}{x-5} \quad x \neq \pm 5 \text{ or } (-\infty, -5) \cup (-5, 5) \cup (5, \infty) \\
fg &= \frac{6x^2-10x-4}{x^2-25} \quad x \neq \pm 5 \text{ or } (-\infty, -5) \cup (-5, 5) \cup (5, \infty) \\
\frac{f}{g} &= \frac{3x+1}{2x-4} \quad x \neq \pm 5 \text{ or } (-\infty, -5) \cup (-5, 2) \cup (2, 5) \cup (5, \infty)
\end{align*}
\]

4. \( f(x) = \sqrt{x+4} \) and \( g(x) = \sqrt{x-1} \)

\[
\begin{align*}
f + g &= \sqrt{x+4} + \sqrt{x-1} \quad x \geq 1 \text{ or } [1, \infty) \\
f - g &= \sqrt{x+4} - \sqrt{x-1} \quad x \geq 1 \text{ or } [1, \infty) \\
fg &= \sqrt{x^2+3x-4} \quad x \geq 1 \text{ or } [1, \infty) \\
\frac{f}{g} &= \frac{\sqrt{x+4}}{\sqrt{x-1}} \quad x > 1 \text{ or } (1, \infty)
\end{align*}
\]

Find \( (f \circ g)(x) \), \( (g \circ f)(x) \), and \( (f \circ g)(2) \) and its domain of \( (f \circ g)(x) \) for the following pairs of functions:

1. \( f(x) = 4x-3 \) and \( g(x) = 5x^2 - 2 \)

\( (f \circ g)(x) = 20x^2 - 11 \) All Real Numbers

\( (g \circ f)(x) = 80x^2 - 120x + 43 \)

\( (f \circ g)(2) = 69 \)

2. \( f(x) = x^2 + 2 \) and \( g(x) = x^2 - 2 \)

\( (f \circ g)(x) = x^4 - 4x^2 + 6 \) All Real Numbers

\( (g \circ f)(x) = x^4 + 4x^2 + 2 \)

\( (f \circ g)(2) = 6 \)

3. \( f(x) = 4-x \) and \( g(x) = 2x^2 + x + 5 \)

\( (f \circ g)(x) = -2x^2 - x - 1 \) All Real Numbers

\( (g \circ f)(x) = 2x^2 - 17x + 41 \)

\( (f \circ g)(2) = -11 \)
4. \( f(x) = \frac{2}{x+3} \) and \( g(x) = \frac{1}{x} \)

\[(f \circ g)(x) = \frac{2x}{1+3x}, \quad x \neq -\frac{1}{3}, 0 \text{ or } (-\infty, -1/3) \cup (-1/3, 0) \cup (0, \infty)\]

\[(g \circ f)(x) = \frac{x+3}{2}\]

\[(f \circ g)(2) = \frac{4}{7}\]

5. \( f(x) = \sqrt{x} \) and \( g(x) = x - 2 \)

\[(f \circ g)(x) = \sqrt{x-2}, \quad x \geq 2 \text{ or } [2, \infty)\]

\[(g \circ f)(x) = \sqrt{x} - 2\]

\[(f \circ g)(2) = 0\]

**Section 2.7**

Find the inverse of each of the following

1. \( f(x) = 2x + 3 \) \( \quad f^{-1}(x) = \frac{x-3}{2} \)
2. \( f(x) = x^3 + 2 \) \( \quad f^{-1}(x) = \sqrt[3]{x-2} \)
3. \( f(x) = \sqrt{x} \) \( \quad f^{-1}(x) = x^2, x \geq 0 \)
4. \( f(x) = \frac{7}{x} - 3 \) \( \quad f^{-1}(x) = \frac{7}{x+3} \)

**Section 2.8**

Find the distance and midpoint between each pair of points

1. \((2, 3)\) and \((14, 8)\) d = 13, m = \((8, 11/2)\)
2. \((-2, -6)\) and \((3, -4)\) d = \(\sqrt{29}\), m = \((1/2, -5)\)
3. \((0, -3)\) and \((4, 1)\) d = \(4\sqrt{2}\), m = \((2, -1)\)

Write the standard form of the equation of the circle with the given center and radius

1. \((0, 0)\) and \(r = 7\) \(x^2 + y^2 = 49\)
2. \((-1, 4)\) and \(r = 2\) \((x+1)^2 + (y-4)^2 = 4\)
3. \((-5, -3)\) and \(r = \sqrt{5}\) \((x+5)^2 + (y+3)^2 = 5\)

If the equation of the circle is not in standard form then write it in standard for and find the center, radius, domain, range, and graph see book for graphs

1. \(x^2 + y^2 = 16\) \(c(0, 0), r = 4, D: [-4, 4], R: [-4,4]\) #41
2. \((x-3)^2 + (y-1)^2 = 36\) \(c(3, 1), r = 6, D: [-3, 9], R: [-5, 7]\) #43
3. \((x+3)^2 + (y-2)^2 = 4\) \(c(-3, 2), r = 2, D: [-5, -1], R: [0, 4]\) #45
4. \(x^2 + y^2 + 6x + 2y + 6 = 0\) \(c(-3, -1), r = 2, D: [-5, -1], R: [-3, 1]\) #49
5. \(x^2 + y^2 + 8x - 2y - 8 = 0\) \(c(-4, 1), r = 5, D: [-9, 1], R: [-4, 6]\) #53
6. \(x^2 + y^2 - 6y - 7 = 0\) \(c(0, 3), r = 4, D: [-4, 4], R: [-1, 7]\)

Section 3.1

Find the vertex, x-intercepts, y-intercepts, equation for the axis of symmetry, and graph each of the following See book for graphs

1. \(f(x) = (x-4)^2 - 1\) #17
   \(v(4, -1), x\)-int 5 and 3, y-int 15, \(x = 4\)
2. \(f(x) = (x-1)^2 + 2\) #19
   \(v(1, 2), x\)-int none, y-int 3, \(x = 1\)
3. \(f(x) = x^2 - 2x - 3\) #27
   \(v(1, -4), x\)-int -1 and 3, y-int -3, \(x = 1\)
4. \(f(x) = x^2 + 6x + 3\) #33
   \(v(-3, -6), x\)-int -3+√6 , y-int 3, \(x = -3\)
5. \(f(x) = 2x - x^2 - 2\) #37
   \(v(1, -1), x\)-int none, y-int -2, \(x = 1\)

Determine without graphing weather the function had a maximum or a minimum. Find the maximum or minimum value and determine where it occurs and identify the domain and range.

1. \(f(x) = 3x^2 - 12x - 1\) min at \((2, -13), D: (-\infty, \infty), R: (-\infty, 0]\)
2. \(f(x) = -4x^2 + 8x - 3\) max at \((1, 1), D: (-\infty, \infty), R: (\infty, 1]\)

Section 3.2

For each function given below do the following See book for answers

a) Use the leading coefficient test to determine the graph’s end behavior
b) Find the x-intercepts and state whether the graph crosses the x-axis or touches the x-axis and turns around
c) Find the y-intercept
d) Determine whether the graph has y-axis, origin symmetry or neither.
e) If necessary, find a few additional points and graph the function. Use the maximum number of turning points to check whether it is drawn correctly.

1. \(f(x) = x^3 + 2x^2 - x - 2\) #41
2. \(f(x) = x^4 - 9x^2\) #43
3. \(f(x) = 6x^3 - 9x - x^5\) #51
4. \( f(x) = x^4 - 2x^3 + x^2 \) #47

Section 3.3

Divide using long division

1. \((x^3 + 5x^2 + 7x + 2) \div (x + 2)\) \(x^2 + 3x + 1\)
2. \((6x^3 + 7x^2 + 12x - 5) \div (3x - 1)\) \(2x^2 + 3x + 5\)
3. \(\frac{x^4 - 81}{x - 3}\) \(x^3 + 3x^2 + 9x + 27\)

Divide using synthetic division

1. \((4x^3 - 3x^2 + 3x - 1) \div (x - 1)\) \(4x^2 + x + 4 + \frac{3}{x - 1}\)
2. \((x^2 - 5x - 5x^3 + x^4) \div (5 + x)\) \(x^3 - 10x^2 + 51x - 260 + \frac{1300}{x + 5}\)
3. \(\frac{x^4 - 256}{x - 4}\) \(x^3 + 4x^2 + 16x + 64\)

- For \(f(x) = 4x^3 + 5x^2 - 6x - 4\) find \(f(-2)\) using the remainder theorem \(f(-2) = -4\)
- Solve the equation \(2x^3 - 3x^2 - 11x + 6 = 0\) given that -2 is a zero of \(f(x) = 2x^3 - 3x^2 - 11x + 6\) \(x = -2, \frac{1}{2}, 3\)

Section 3.4

Find all the zeros of the following polynomial functions

1. \(f(x) = x^3 - 2x^2 - 11x + 12\) \(x = -3, 1, 4\)
2. \(f(x) = 2x^3 + 6x^2 + 5x + 2\) \(x = -2, -\frac{1}{2} \pm \frac{1}{2}i\)
3. \(f(x) = x^4 - 2x^3 - 5x^2 + 8x + 4\) \(x = \pm 2, 1 \pm \sqrt{2}\)

Find the nth degree polynomial function with real coefficients satisfying the given conditions.

1. \(n = 3\); 1 and 5i are zeros; \(f(-1) = -104\) \(f(x) = 2x^3 - 2x^2 + 50x - 50\)
2. \(n = 4\); i and 3i are zeros ; \(f(-1) = 20\) \(f(x) = x^4 + 10x^2 + 9\)
3. \( n = 3; \) \(-5\) and \(4+3i\) are zeros; \( f(2) = 91 \)

\[ f(x) = x^3 - 3x^2 - 15x + 125 \]