Homework 3 Solution (EE2111)

Q1)

16. Given the excitations, $\mathrm{x}[n]$, and the impulse responses, $\mathrm{h}[n]$, use MATLAB to plot the system responses, $\mathrm{y}[n]$.

(a) $\mathrm{x}[n]=\mathrm{u}[n]-\mathrm{u}[n-8]$, $\mathrm{h}[n]=\sin\left(\frac{2\pi n}{8}\right)(\mathrm{u}[n]-\mathrm{u}[n-8])$

$$\mathrm{y}[n]=\mathrm{h}[n]*\mathrm{x}[n]=\sum_{m=-\infty}^{\infty}\sin\left(\frac{2\pi m}{8}\right)(\mathrm{u}[m]-\mathrm{u}[m-8])(\mathrm{u}[n-m]-\mathrm{u}[n-m-8])$$

$$\mathrm{y}[n]=\sum_{m=-\infty}^{\infty}\sin\left(\frac{2\pi m}{8}\right)\begin{pmatrix}\mathrm{u}[m]\mathrm{u}[n-m]-\mathrm{u}[m]\mathrm{u}[n-m-8]\\ -\mathrm{u}[m-8]\mathrm{u}[n-m]+\mathrm{u}[m-8]\mathrm{u}[n-m-8]\end{pmatrix}$$

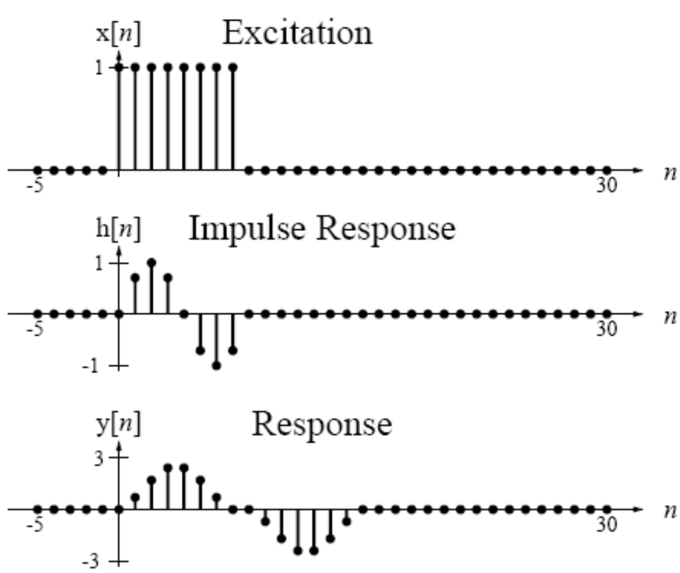
$$\mathrm{y}[n]=\sum_{m=0}^{n}\sin\left(\frac{2\pi m}{8}\right)-\sum_{m=0}^{n-8}\sin\left(\frac{2\pi m}{8}\right)-\sum_{m=8}^{n}\sin\left(\frac{2\pi m}{8}\right)+\sum_{m=8}^{n-8}\sin\left(\frac{2\pi m}{8}\right)$$

For $n<0$, all the summations are zero because the factors, $\mathrm{u}[n-m]$ and $\mathrm{u}[n-m-8]$ are zero in the summation range, $0<m<n$, and $\mathrm{y}[n]=0$.

For $n>15$,

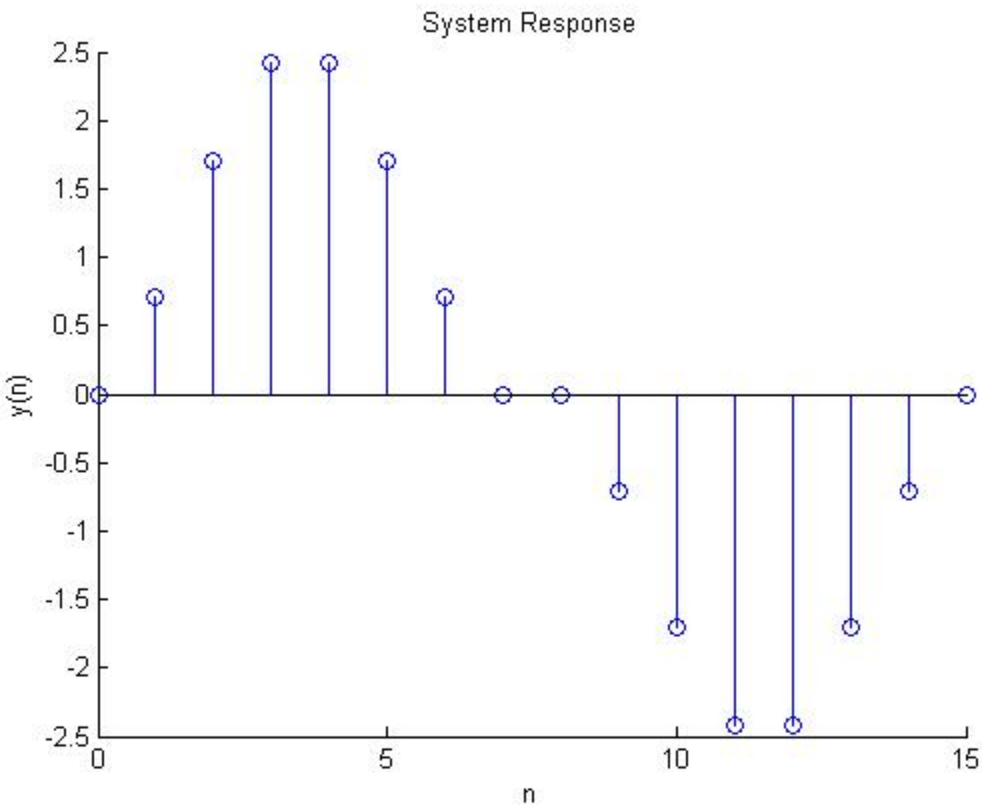
$$\mathrm{y}[n]=\sum_{m=n-7}^{n}\sin\left(\frac{2\pi m}{8}\right)-\sum_{m=n-7}^{n}\sin\left(\frac{2\pi m}{8}\right)=0\ .$$

So the response is only non-zero for $0\le m<16$ (and can be zero at some points within that range).



MATLAB code for the part (a)

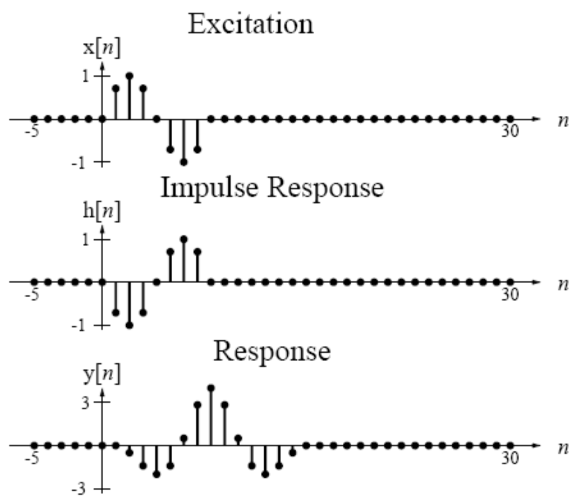
```
t = 1;
n = 1:t:8;
x(n) = 1
l1 = length(x)
h(n) = sin((2*pi*(n-1))/8)
y = conv(x,h)
n = 1:t:16;
stem(y)
```



(b) $\quad \mathrm{x}[n]=\sin\left(\frac{2\pi n}{8}\right)\left(\mathrm{u}[n]-\mathrm{u}[n-8]\right) \quad , \quad \mathrm{h}[n]=-\sin\left(\frac{2\pi n}{8}\right)\left(\mathrm{u}[n]-\mathrm{u}[n-8]\right)$

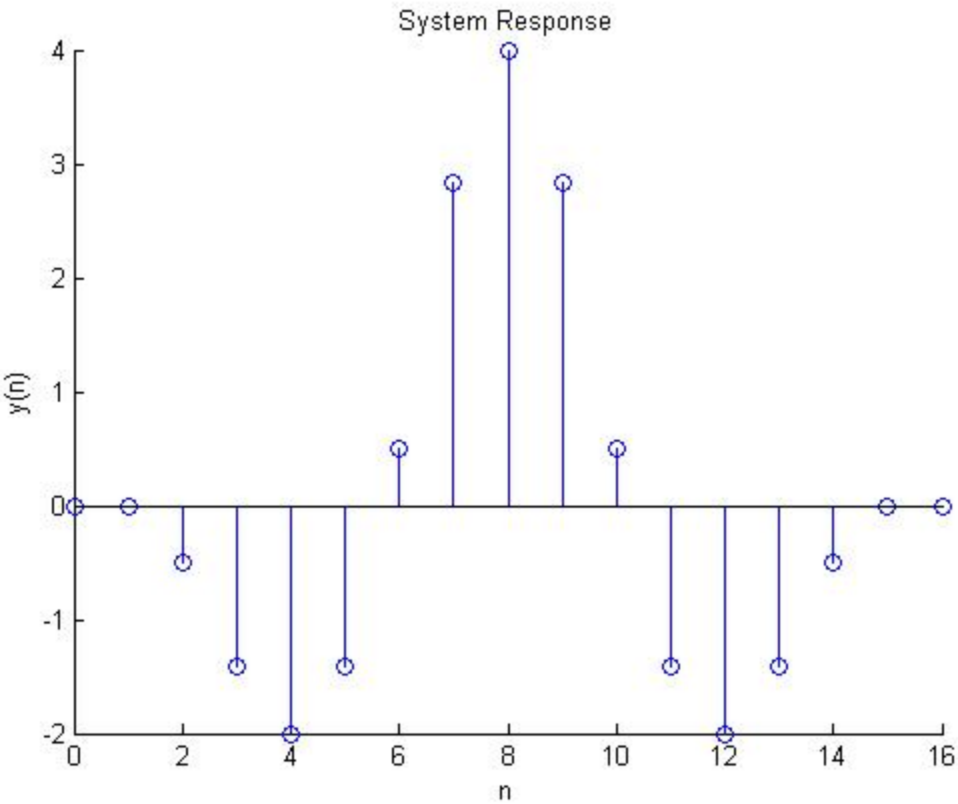
$$\mathrm{y}[n]=\mathrm{h}[n]*\mathrm{x}[n]=\sum_{m=-\infty}^{\infty}-\sin\left(\frac{2\pi m}{8}\right)\left(\mathrm{u}[m]-\mathrm{u}[m-8]\right)\sin\left(\frac{2\pi(n-m)}{8}\right)\left(\mathrm{u}[n-m]-\mathrm{u}[n-m-8]\right)$$

$$\mathrm{y}[n]=-\left\{\begin{array}{l}\sum_{m=0}^{n}\sin\left(\frac{2\pi m}{8}\right)\sin\left(\frac{2\pi(n-m)}{8}\right)-\sum_{m=8}^{n}\sin\left(\frac{2\pi m}{8}\right)\sin\left(\frac{2\pi(n-m)}{8}\right)\\ -\sum_{m=0}^{n-8}\sin\left(\frac{2\pi m}{8}\right)\sin\left(\frac{2\pi(n-m)}{8}\right)+\sum_{m=8}^{n-8}\sin\left(\frac{2\pi m}{8}\right)\sin\left(\frac{2\pi(n-m)}{8}\right)\end{array}\right\}$$



MATLAB code for part (b)

```
t = 1;
n = 1:t:9;
x(n) = sin((2*pi*(n-1))/8);;
h(n) = -sin((2*pi*(n-1))/8);
y = conv(x,h);
n = 1:t:17
stem(y)
xlabel('n'),ylabel('y(n)'),title('System Response')
```

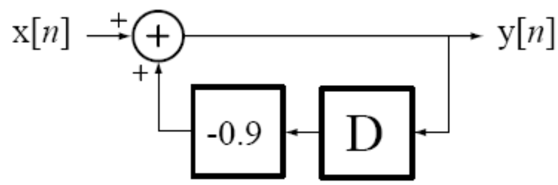


System Response
y(n)
n
4
3
2
1
0
-1
-2
0
2
4
6
8
10
12
14
16

Q2)

17. Which of these systems are BIBO stable?

(a)

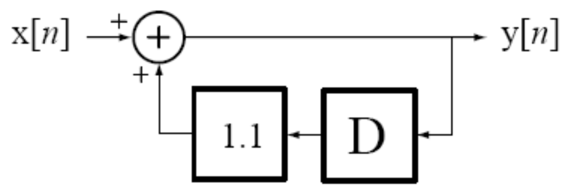


The system equation is

$$y[n] = x[n] - 0.9\,y[n-1]$$

If the unit impulse is applied at the input (bounded input), the output will remain bounded and not growing with time because the output at any given n is the difference of input and previous output multiplied by a number less than 1.

(b)

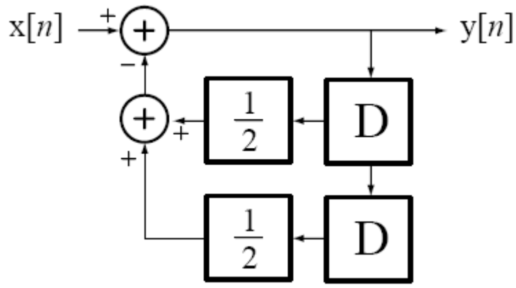


The system equation is

$$y[n] = x[n] + 1.1\,y[n-1]$$

If the unit impulse is applied at the input (bounded input), the output will not remain bounded and will grow with time because the output at any given n is the sum of input and previous output multiplied by a number more than 1.

(c)



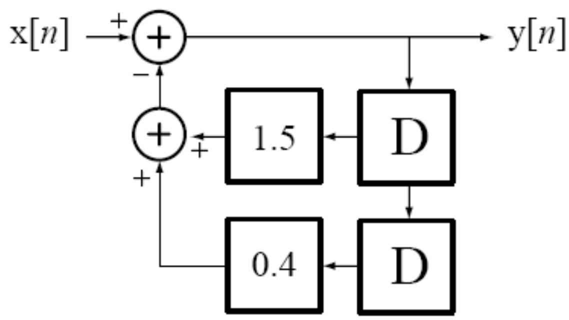
The system equation is

$$y[n] = x[n] - \frac{1}{2}\left(y[n-1] + y[n-2]\right)$$

The system in stable with the same reasoning as in (a).

(d)

System is unstable with the same reason as in (b)



The system equation is

$$y[n] = x[n] - \left(1.5\,y[n-1] + 0.4\,y[n-2]\right)$$

Q3)

20. Sketch $\mathrm{g}(t)$.

(a) $$\mathrm{g}(t)=\mathrm{rect}(t)*\mathrm{rect}(t)=\int_{-\infty}^{\infty}\mathrm{rect}(\tau)\mathrm{rect}(t-\tau)\,d\tau=\int_{-\frac{1}{2}}^{\frac{1}{2}}\mathrm{rect}(t-\tau)\,d\tau$$

Probably the easiest way to find this solution is graphically through the "flipping and shifting" process. When the second rectangle is flipped, it looks exactly the same because it is an even function. This is the "zero shift" position, the $t=0$ position. At this position the two rectangles coincide and the area under the product is one. If t is increased from this position the two rectangles no longer coincide and the area under the product is reduced linearly until at $t=1$ the area goes to zero. Exactly the same thing happens for decreases in t until it gets to -1. The convolution is obviously a unit triangle function. This fact is the reason the unit triangle function was defined as it was, so it could simply be the convolution of a unit rectangle with itself.

This convolution can also be done analytically.

For $t<-1$, in the range of integration, $-\frac{1}{2}<\tau<\frac{1}{2}$, the rect function is zero and the convolution integral is zero.

For $t>1$, in the range of integration, $-\frac{1}{2}<\tau<\frac{1}{2}$, the rect function is zero and the convolution integral is zero.

For $-1<t<0$. Since the rect function is even we can say that $\mathrm{rect}(t-\tau)=\mathrm{rect}(\tau-t)$. This is a rectangle extending in τ from $t-\frac{1}{2}$ to $t+\frac{1}{2}$. For t's in the range, $-1<t<0$, $t-\frac{1}{2}$ is always less than or equal to the lower limit, $\tau=-\frac{1}{2}$, so the integral is from $-\frac{1}{2}$ to $t+\frac{1}{2}$.

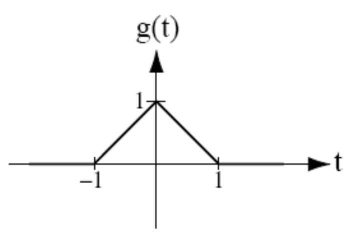
$$g(t) = \int_{-\frac{1}{2}}^{t+\frac{1}{2}} \mathrm{rect}(\tau - t)\, d\tau$$

This is simply the accumulation of the area under a rectangle and therefore increases linearly from a minimum of zero for $t = -1$ to a maximum of one for $t = 0$.

For $0 < t < 1$. This is also rectangle extending in τ from $t - \frac{1}{2}$ to $t + \frac{1}{2}$. For t's in the range, $0 < t < 1$, $t + \frac{1}{2}$ is always greater than or equal to the upper limit, $\tau = \frac{1}{2}$, so the integral is from $t - \frac{1}{2}$ to $\frac{1}{2}$.

$$g(t) = \int_{t-\frac{1}{2}}^{\frac{1}{2}} \mathrm{rect}(\tau - t)\, d\tau$$

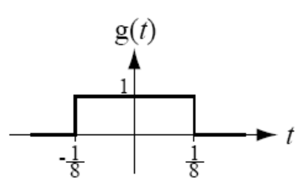
This is also the accumulation of the area under a rectangle and decreases linearly from a maximum of one for $t = 0$ to a minimum of zero for $t = 1$.



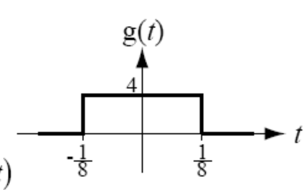
Q4)

21. Sketch these functions.

(a) $g(t) = \mathrm{rect}(4t)$



(b) $g(t) = \mathrm{rect}(4t) * 4\delta(t)$



Q5)

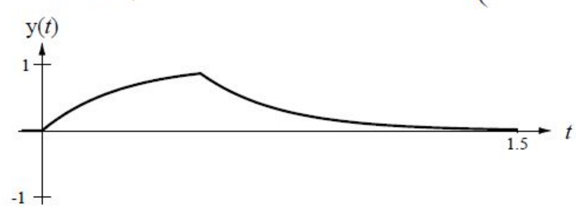
23. A system has an impulse response, $h(t) = 4e^{-4t}u(t)$. Find and plot the response of the system to the excitation, $x(t) = \text{rect}\left(2\left(t - \frac{1}{4}\right)\right)$.

$$y(t) = x(t) * h(t) = \text{rect}\left(2\left(t - \frac{1}{4}\right)\right) * 4e^{-4t}u(t) = 4\left(u(t) - u\left(t - \frac{1}{2}\right)\right) * e^{-4t}u(t)$$

$$u(t) * e^{-4t}u(t) = \int_{-\infty}^{\infty} u(\tau)e^{-4(t-\tau)}u(t-\tau)d\tau = \int_0^t e^{-4(t-\tau)}d\tau = \begin{Bmatrix} \frac{1}{4}\left(1 - e^{-4t}\right) & , t > 0 \\ 0 & , t < 0 \end{Bmatrix} = \frac{1}{4}\left(1 - e^{-4t}\right)u(t)$$

Invoking linearity and time-invariance,

$$y(t) = 4\left(u(t) - u\left(t - \frac{1}{2}\right)\right) * e^{-4t}u(t) = \left(1 - e^{-4t}\right)u(t) - \left(1 - e^{-4\left(t - \frac{1}{2}\right)}\right)u\left(t - \frac{1}{2}\right)$$

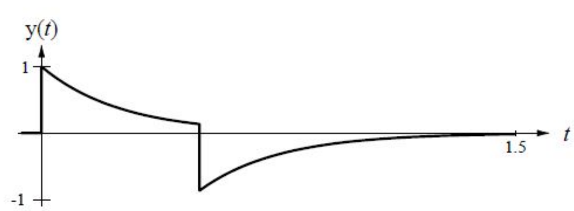


Q6)

24. Change the system impulse response in Exercise 23 to $h(t) = \delta(t) - 4e^{-4t}u(t)$ and find and plot the response to the same excitation, $x(t) = \text{rect}\left(2\left(t - \frac{1}{4}\right)\right)$.

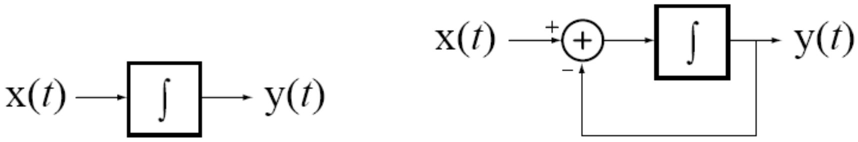
Using linearity, this response is the excitation convolved with a unit impulse at time zero minus the response calculated in Exercise 23.

$$y(t) = \text{rect}\left(2\left(t - \frac{1}{4}\right)\right) - \left(1 - e^{-4t}\right)u(t) - \left(1 - e^{-4\left(t - \frac{1}{2}\right)}\right)u\left(t - \frac{1}{2}\right)$$



Q7)

25. Find the impulse responses of the two systems in Figure E25. Are these systems BIBO stable?



(a) (b)

Figure E25 Two single-integrator systems

(a) $y'(t) = x(t) \Rightarrow h(t) = u(t)$

Impulse response is not absolutely integrable. BIBO unstable.

(b) $y'(t) = x(t) - y(t) \Rightarrow h(t) = e^{-t}u(t)$ BIBO Stable.

Impulse response is absolutely integrable. BIBO stable.

Q8)

43. Find the impulse response, $h[n]$, of the system in Figure E43.

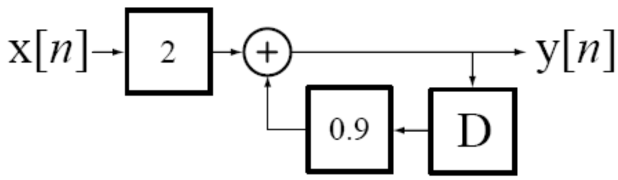


Figure E43 DT system block diagram

$$y[n] = 2x[n] + 0.9y[n-1]$$

or

$$y[n] - 0.9y[n-1] = 2x[n]$$

The homogeneous solution (for $n \geq 0$) is of the form,

$$y[n] = K_h \alpha^n$$

We can find an initial condition to evaluate the constant, K_h, by directly solving the difference equation for $n = 0$.

$$y[0] = 2x[0] + 0.9y[-1] = 2 .$$

Therefore

$$2 = K_h (0.9)^0 \Rightarrow K_h = 2 .$$

Therefore the total solution is

$$y[n] = 2(0.9)^n$$

IMORTANT NOTE: The preferred method will be recursive method because even for complicated systems, you can find impulse response easily.

Q9)

a) $e^{-t}u(t)$
b) $(1\text{-}e^{-t})u(t)$
c) $(1\text{-}e^{-(t-1)})u(t-1)$
d) $(1\text{-}e^{-t})u(t) - (1\text{-}e^{-(t-1)})u(t-1)$

Q10)

a) $e^{-2t}u(t)$
b) $(1\text{-}e^{-2t})u(t)$
c) $(1\text{-}e^{-(2(t-1))})u(t-1)$
d) $(1\text{-}e^{-2t})u(t) - (1\text{-}e^{-(2(t-1))})u(t-1)$