Integrate the following:

1. $\int \sin^4 x \cos^3 x dx$

$\int \sin^4 x \cos^2 x \cos x \, dx \Rightarrow \int \sin^4 x(1-\sin^2 x) \cos x \, dx \quad (u=\sin x, du=\cos x dx)$

$\int u^4(1-u^2) du \Rightarrow \int u^4 - u^6 du \Rightarrow \frac{1}{5}u^5 - \frac{1}{7}u^7 + C \Rightarrow \boxed{\frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + C}$

2. $\int \sqrt{\tan x} \sec^4 x dx$

$\int \tan^{1/2} x \sec^2 x \sec^2 x \, dx \Rightarrow \int \tan^{1/2} x (1+\tan^2 x) \sec^2 x \, dx \quad (u=\tan x \;\; du=\sec^2 x dx)$

$\int u^{1/2}(1+u^2) du \Rightarrow \int u^{1/2} + u^{5/2} du \Rightarrow \frac{2}{3}u^{3/2} + \frac{2}{7}u^{7/2} + C \Rightarrow$

$\boxed{\frac{2}{3}(\tan x)^{3/2} + \frac{2}{7}(\tan x)^{7/2} + C}$

3. $\int \frac{1}{1+\cos 2x} dx$

table

$(1+\cos(2x) = 2\cos^2 x) \Rightarrow \int \frac{1}{2\cos^2 x} dx \Rightarrow \frac{1}{2}\int \sec^2 x \, dx \Rightarrow$

$\boxed{\frac{1}{2}(\tan x) + C}$

4. $\int \frac{1}{\sqrt{25-x^2}} dx$ table or trig sub

$\int \frac{1}{\sqrt{5^2 - x^2}} dx \Rightarrow \sin^{-1}\left(\frac{x}{5}\right) + C$

5. $\int \frac{1}{\sqrt{1-x-x^2}}dx$ complete the square, u sub.

$= \sin^{-1}\left(\frac{1+2x}{\sqrt{5}}\right)$

6. $\int x^2\sqrt{4-x^2}\,dx$

table $\Rightarrow \frac{x}{8}(2x^2-4)\sqrt{4-x^2} + \frac{2^4}{8}\sin^{-1}\left(\frac{x}{2}\right)+C$

7. $\int \frac{2x+1}{x^2+3x+2}dx$

$\frac{2x+1}{(x+2)(x+1)} = \frac{A}{(x+2)} + \frac{B}{(x+1)} \Rightarrow 2x+1 = A(x+1)+B(x+2) \Rightarrow 2x+1 = Ax+A+Bx+2B \Rightarrow$

$2x+1 = (A+B)x + A+2B$

$A+B=2 \to A = 2-B$

$A+2B=1 \to 2-B+2B=1 \to \underline{B=-1}, \underline{A=3}$

$3\int \frac{1}{x+2}dx - \int \frac{1}{x+1}dx \Rightarrow \boxed{3\ln(x+2) - \ln(x+1) \quad +C}$

8. $\int \frac{2x+1}{x^2+2x+2}dx$

$-\tan^{-1}(1+x) + \ln(2+2x+x^2)$

9. $\int \frac{2x^4+2}{x^2+3x+2}dx$

$$\frac{30x+26}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2} \Rightarrow 30x+26 = A(x+2)+B(x+1)$$

$$30x+26 = (A+B)x + 2A + B$$

$A+B = 30 \qquad B = 30 - A$

$2A+B = 26$

$2A + 30 - A = 26 \Rightarrow A = -4$

$B = 34$

$$2x^2 - 6x + 14 - \boxed{\frac{30x+26}{x^2+3x+2}}$$

$$\begin{array}{r} x^2+3x+2 \;\big)\; 2x^4+0x^3+0x^2+0x+2 \\ -\;(2x^4+6x^3+4x^2) \\ \hline -6x^3-4x^2+2 \\ -\;(-6x^3-18x^2-12x) \\ \hline 14x^2+12x+2 \\ -\;(14x^2+42x+28) \\ \hline -30x-26 \end{array}$$

$$\boxed{\frac{2}{3}x^3 - 3x^2 + 14x + 4\ln(x+1) - 34\ln(x+2)}$$

10. $\int \frac{2x+1}{2x^3+6x^2+4x}dx$

$$\frac{2x+1}{x(2x+4)(x+1)} = \frac{A}{x} + \frac{B}{2x+4} + \frac{C}{x+1} \Rightarrow 2x+1 = A(2x^2+6x+4) + B(x^2+x) + C(2x^2+4x)$$

$$2x+1 = 2Ax^2 + 6Ax + 4A + Bx^2 + Bx + 2Cx^2 + 4Cx$$

$$2x+1 = (2A+B)x^2 + (6A+B+4C)x + 4A$$

$2A+B=0 \qquad A = 1/4$

$6A+B+4C=2 \qquad B = -.5$

$4A = 1 \qquad C = .25$

$$\boxed{\frac{1}{4}\ln(x) - \frac{1}{4}\ln(2x+4) + \frac{1}{4}\ln(x+1)}$$

11. $\int \frac{2}{(x^2-x-6)(x^2+6x+9)}dx$

$$\frac{2}{(x-3)(x+2)(x+3)^2} = \frac{A}{x-3} + \frac{B}{x+2} + \frac{C}{x+3} + \frac{D}{(x+3)^2}$$

$$2 = A(x+2)(x+3)^2 + B(x-3)(x+3)^2 + C(x-3)(x+2)(x+3) + D(x-3)(x+2)$$

$x=-2 \Rightarrow 2 = -B5 \Rightarrow B = -2/5$

$x=-3 \Rightarrow 2 = 6D \Rightarrow D = 1/3$

$x=3 \Rightarrow 2 = 180A \Rightarrow A = 1/90$

$C = -\frac{1}{3}$

$$\frac{1}{90}\ln(x-3) - \frac{2}{5}\ln(x+2) - \frac{7}{18}\ln(x+3) - \frac{1}{3(3+x)}$$

12. $\int \frac{x+2x^2}{x}dx = \int(1+2x)dx = x + x^2 + C$

$v = x \qquad dv = dx$

$du = \sqrt{x-5}\,dx \qquad u = \frac{2}{3}(x-5)^{\frac{3}{2}}$

13. $\int x\sqrt{x-5}dx$

$$= \frac{2x}{3}(x-5)^{\frac{3}{2}} - \int \frac{2}{3}(x-5)^{\frac{3}{2}}\,dx$$

$$= \frac{2x}{3}(x-5)^{\frac{3}{2}} - \frac{2}{3}\cdot\frac{2}{5}(x-5)^{\frac{5}{2}} + C$$

14. $\int \sin x \cos(\cos x)dx$ u sub.

$$= -\sin(\cos x) + C$$

u sub

15. $\int \sin(2x+3)dx = -\frac{1}{2}\cos(2x+3) + C$

16. $\int \frac{x^4}{x^{10}+2}dx$ Let $u = x^{10}+2 \Rightarrow x^5 = \sqrt{u-2}$

$du = 10x^9\,dx$

$$= \int \frac{x^9\,dx}{x^5(x^{10}+2)} = \frac{1}{10}\int \frac{du}{u\sqrt{u-2}} = \frac{1}{10}\cdot\frac{2}{\sqrt{2}}\tan^{-1}\left(\sqrt{\frac{u-2}{2}}\right) + C$$

by table 51

$$= \frac{1}{5\sqrt{2}}\tan^{-1}\left(\sqrt{\frac{x^{10}}{2}}\right) + C$$

table entry

17. $\int x^2\sqrt{5-x^2}\,dx$ (table) $\quad \frac{x}{8}(2x^2-5)\sqrt{5-x^2} + \frac{25}{8}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)+C$

table entry or reduction formula or trig sub.

18. $\int \tan^5 x\,dx$ (table)

$$= \frac{1}{5-1}\tan^4 x - \int \tan^3 x\,dx = \frac{1}{4}\tan^4 x - \left(\frac{1}{2}\tan^2 x - \int \tan x\,dx\right)$$

$$= \frac{1}{4}\tan^4 x - \frac{1}{2}\tan^2 x + \ln|\sec x| + C$$

5y #12

never mind this

Use the left, right midpoint, trapezoidal, and Simpson's method to estimate the following areas. Use an adequate number of intervals to estimate the area to be within .01 of the actual area (use error formulas – they will be given on the real exam)

19. $\int_1^5 \ln x\,dx$

$.01 \ge \frac{(4)^3}{12n^2} \Rightarrow .12n^2 \ge 64 \Rightarrow n \ge 24$

$T_{24} = \frac{1}{12}\left(f(1) + 2f(1\tfrac{1}{6}) + \cdots + f(5)\right) = 4.045$

$.01 \ge \frac{6(4)^5}{180n^4} \Rightarrow 1.8n^4 \ge 6144 \Rightarrow n \ge 8$

$S_8 = \frac{1}{6}\left(f(1) + 4f(1.5) + \cdots + f(5)\right) \Rightarrow 4.046$

$.01 \ge \frac{4^3}{24n^2} \Rightarrow n \ge 17$

$M_4 = \frac{4}{17}\left[f(\tfrac{19}{17}) + f(\tfrac{23}{17}) + \cdots + f(\tfrac{83}{17})\right] = 4.049$

$n = 32$

$L_4 =$

20. $\int_0^1 \cos(\tan x)\,dx$

will post separately