Math 3280 Numerical methods/IVP review.

(a) Solve the initial value problem $x'' + 2x' + 5x = t$, $x(0) = 0$, $x'(0) = 0$, and find the exact value of $x(2)$.

(b) Approximate the solution $x(2)$ using two steps of the Improved Euler method. (Optional but recommended: compare this with one step of the Runge-Kutta fourth order method.)

The Improved Euler method is:

$$k_1 = f(t_n, \vec{x}_n)$$
$$k_2 = f(t_n + h, \vec{x}_n + hk_1)$$

$$\vec{x}_{n+1} = \vec{x}_n + h(k_1 + k_2)/2$$
$$t_{n+1} = t_n + h$$

For this worksheet, $\vec{x} = (x, v)$ where $v = x'$. Each slope k_i is also a two-component vector, and the function f is vector-valued (i.e. it outputs a two-component vector).

The 4th-order Runge-Kutta formulae are:

$$k_1 = f(t_n, \vec{x}_n)$$
$$k_2 = f(t_n + h/2, \vec{x}_n + hk_1/2)$$
$$k_3 = f(t_n + h/2), \vec{x}_n + hk_2/2)$$
$$k_4 = f(t_n + h, \vec{x}_n + hk_3)$$

Finally

$$\vec{x}_{n+1} = \vec{x}_n + h(k_1 + 2k_2 + 2k_3 + k_4)/6$$
$$t_{n+1} = t_n + h$$