Math 3280 Assignment 11, due Friday, November 30th.

The problems in this assignment are primarily based on sections 6.1, 6.2, 6.3, 7.1, and 7.3 in the text.

Find the eigenvalues and eigenvectors of the following matrices:

(1) $\begin{pmatrix} 4 & -2 \\ 1 & 1 \end{pmatrix}$

(2) $\begin{pmatrix} 5 & -6 \\ 3 & -4 \end{pmatrix}$

(3) $\begin{pmatrix} 2 & 0 & 0 \\ 5 & 3 & -2 \\ 2 & 0 & 1 \end{pmatrix}$

(4) $\begin{pmatrix} 3 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(5) $\begin{pmatrix} 0 & -2 \\ 1 & 0 \end{pmatrix}$

(6) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Find a matrix P such that $P^{-1}AP = D$, where D is a diagonal matrix, for the following matrices if such a P exists.

(7) $\begin{pmatrix} 0 & 1 & 0 \\ -1 & 2 & 0 \\ -1 & 1 & 1 \end{pmatrix}$

(8) $\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 \end{pmatrix}$

(9) Show that if A is invertible and λ is an eigenvalue of A, then $1/\lambda$ is an eigenvalue of A^{-1}. Are the eigenvectors the same?

(10) By computing the eigenvalues and eigenvectors of $A = \begin{pmatrix} 3 & -2 \\ 1 & 0 \end{pmatrix}$ find a matrix P such that $P^{-1}AP = D$ where D is a diagonal matrix. Use this diagonalization to compute A^6.

(11) Rewrite the second-order differential equation $x'' + 3x' + 5x = t$ as a system of first-order differential equations. (You do not have to find the solution.)

(12) Find the general solution to the system $x_1' = x_1 + 2x_2$, $x_2' = 2x_1 + x_2$. Sketch some of the solutions near the origin, including some that start on the lines spanned by the eigenvectors of the coefficient matrix of the system.

(13) Find the general solution to the system $x_1' = x_1 + 2x_2$, $x_2' = 3x_1 + 2x_2$.

(14) Find the general solution to the system $x_1' = x_1 - 5x_2$, $x_2' = x_1 - x_2$. Sketch some of the solutions near the origin.

(15) Solve the initial value problem $x_1' = x_1 + 2x_2$, $x_2' = -2x_1 + x_2$, $x_1(0) = 1$, $x_2(0) = 0$.