Math 3280 Assignment 2, due Friday, September 14th.

In addition to the problems below, you should read sections 1.3, 1.4, and 1.5 in our text.

Find the general solutions $y(x)$ to the following separable equations:

(1) $y' = 4xy$.

(2) $(2 + 2x)y' = 4y$.

(3) $y' = y\cos(x)$.

(4) $y' = 1 + x + y + xy$.

Find the solution $y(x)$ to the following initial value problems:

(5) $y' = 2ye^x, \;\; y(0) = 2e^2$.

(6) $y' = x^3(y^2 + 1), \;\; y(0) = 1$.

(7) In carbon-dating organic material it is assumed that the amount of carbon-14 (^{14}C) decays exponentially ($\frac{d\,^{14}C}{dt} = -k\,^{14}C$) with rate constant of $k \approx 0.0001216$ where t is measured in years. Suppose an archeological bone sample contains 1/7 as much carbon-14 as is in a present-day sample. How old is the bone?

(8) In this exercise you will work out a simplified alternate landing scenario for the Mars Science Laboratory spacecraft. You may wish to use a programmable calculator, Sage, or some other computational tool since the computations are unwieldy to do by hand.

Suppose the spacecraft is approaching Mars at a speed of $-470\ m/s$ at time $t = 0$, and at that time it is 25 km above the surface. We will ignore air resistance, so assume that the acceleration of the spacecraft is $-3.7278\ m/s^2$ (Mars surface gravitational acceleration).

(a) Write down the velocity and position of the spacecraft as functions of time.

(b) Now suppose that at time $t = t_f$ the spacecraft begins to fire rockets that change its total acceleration to $5\ m/s^2$. Compute the value of t_f such that the spacecraft lands on the surface with zero velocity. (Hint: it helps to introduce a time of landing, t_L, such that the height and velocity of the spacecraft are zero at $t = t_L$.)

Determine what the existence and uniqueness theorem (Theorem 1 from Chapter 1.3) guarantees about solutions to the following initial value problems (note that you do not have to find the solutions):

(9) $dy/dx = \sqrt{xy}$, $y(0) = 1$.

(10) $dy/dx = y^{1/3}$, $y(0) = 2$.

(11) $dy/dx = y^{1/3}$, $y(2) = 0$.

(12) $dy/dx = x\ln(y)$, $y(0) = 1$.

Solve the following first-order linear ODEs:

(13) $dy/dx = -2y + 2xe^{-2x}$.

(14) $dy/dx + y\tan(x) = \sin(x)$.