

Math 3280 Assignment 4, due Friday, September 28th.

Read sections 2.1-2.3 and 3.1-3.3 in the text.

- (1) Suppose a population of invasive gophers, P , grows at a rate proportional to the square root of the population¹. Initially the population size is 400 and it is increasing (initially) at a rate of 40 gophers/month. What will the population be after 1 year?
- (2) Suppose a population of 15000 people are susceptible to a contagious disease, and that this disease spreads at a rate that is proportional to the number of infected people times the number of uninfected people. If there are initially 1000 people with the disease, and the number of infections is increasing at a rate of 140/day, how much longer will it take for half the population to be infected?

For the following three differential equations find the equilibria and determine their stability. Then solve the differential equation and use all of this information to sketch some of the typical trajectories.

(3) $y' = y - 5$

(4) $y' = y^2 - 3y$

(5) $y' = (y - 1)^2$

- (6) Consider a fish population that would change according to the logistic equation, $P' = kP(M - P)$ if it were undisturbed. Suppose that these fish will be removed at a rate hP for some $h \geq 0$. If $k = 1$ and $M = 1000$, find the value of h that will maximize the number of removed fish at a stable equilibrium population.

¹This dependence is sometimes used for a geographically expanding population, in which there is an internal population plateaued at its maximum sustainable level surrounded by a thin zone of colonization. The size of the thin zone is proportional to the square root of the internal area.