Math 3280 Assignment 5, due Friday, October 5th.

Read sections 2.3 and 3.1-3.3 in the text.

(1) You jump out of an airplane at a height of 3000 meters. After 20 seconds, you open your parachute. Assume a linear air resistance with a drag acceleration of rv where $r = 0.15$ without that parachute and $r = 1.5$ with the parachute. How long does it take to reach the ground?

(2) Suppose that the drag acceleration was exactly proportional to the square of the velocity, so $\dfrac{dv}{dt} = -kv^2$. If there were no other forces involved, and the intial velocity and position were $v(0) = v_0$, $x(0) = x_0$, how far would a particle travel before coming to rest?

(3) Use the method of elimination to solve the linear system

$$\begin{aligned} x + 3y + z &= 7 \\ 2x + z &= 5 \\ 2x + y - z &= 4. \end{aligned}$$

(4) The second-order differential equation $y'' = -9y$ has a two-parameter general solution $y = A\cos(3x) + B\sin(3x)$. For the initial conditions $y(0) = 4$, $y'(0) = 3$ find the values of A and B.

(5) A linear system of the form:

$$\begin{aligned} a_{11}x + a_{12}y &= 0 \\ a_{21}x + a_{22}y &= 0 \end{aligned}$$

is said to be homogeneous (i.e. when the left hand side is zero). Explain in geometric terms why such a system must either have a unique solution or infinitely many solutions. Find a formula for the unique solution when it exists.

(6) Use elementary row operations to transform the augmented coefficient matrix of the system below into echelon or reduced echelon form. Use this to write down all the solutions to the system.

$$\begin{aligned} x + 2y + -2z &= 3 \\ 2x + 4y - 2z &= 2 \\ x + 3y + 2z &= -1 \end{aligned}$$

(7) Same as above, for the system:

$$\begin{aligned} x + 2y - z &= -1 \\ x + 3y - 2z &= 0 \\ -x - z &= 3 \end{aligned}$$

(8) Under what conditions on the numbers a, b, and c does the following system have a unique solution, infinitely many solutions, or no solutions?

$$\begin{aligned} 2x - y + 3z &= a \\ x + 2y + z &= b \\ 6x + 2y + 8z &= c \end{aligned}$$