Math 3280 Assignment 6, due Thursday, October 11th.

For this assignment you should read sections 3.4 - 3.6 in the Edwards and Penney text.

(1) Reduce the following matrix to row reduced echelon form.

$$\begin{pmatrix} 1 & 3 & -6 \\ 2 & 6 & 7 \\ 3 & 9 & 1 \end{pmatrix}$$

For the following three pairs of matrices, compute whatever products between them are defined.

(2)

$$A = \begin{pmatrix} 1 & -2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 3 & 0 \\ 1 & 2 & 1 \end{pmatrix}$$

(3)

$$A = \begin{pmatrix} 2 & 0 & 1 & 0 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

(4)

$$A = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 2 & 2 \\ -1 & 1 \end{pmatrix}$$

(5) Find a two by two matrix A with all non-zero entries such that $A^2 = 0$.

(6) Find a two by two matrix A with each diagonal element equal to zero such that $A^2 = -I$. (The existence of such matrices is important, as it lets us represent complex numbers by real matrices).

(7) Find the inverse A^{-1} of the following matrix by augmenting with the identity and computing the reduced echelon form.

$$A = \begin{pmatrix} 5 & 3 & 6 \\ 4 & 2 & 3 \\ 3 & 2 & 5 \end{pmatrix}$$

(8) Compute the inverse of A to find a matrix X such that $AX = B$ if $A = \begin{pmatrix} 5 & 3 \\ 3 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 2 & 4 \\ 1 & 6 & -4 \end{pmatrix}$.

(9) Show that if A, B, and C are invertible matrices then $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$.

The next two problems are much easier if you use some of the properties of determinants.

(10) Compute the determinant of $A = \begin{pmatrix} 2 & 3 & 4 & 5 \\ 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 1 \end{pmatrix}$.

(11) Compute the determinant of $A = \begin{pmatrix} 3 & 1 & 0 & 2 \\ 0 & 1 & 20 & 2 \\ 0 & 0 & 1 & -63 \\ 9 & 3 & 0 & 1 \end{pmatrix}$.

(12) Show that if a matrix A has the property $A^2 = A$, then either $det(A) = 0$ or $det(A) = 1$.

(13) A matrix is called orthogonal if $A^T = A^{-1}$. Show that if A is orthogonal then $det(A) = \pm 1$.