

Math 3280 Assignment 7, due Wednesday, October 24th.

This assignment covers material from much of Chapter 4, in particular 4.3, 4.4, and 4.7. We will mostly skip section 4.6.

- (1) Find a quadratic polynomial  $a_2x^2 + a_1x + a_0$  whose graph passes through the points  $(1, 3)$ ,  $(2, 3)$ , and  $(4, 9)$ .
- (2) Determine whether the vectors  $(0, 2)$  and  $(0, 5)$  are linearly dependent or independent.
- (3) Express  $w = (1, 2)$  as a linear combination of  $u = (-1, -1)$  and  $v = (2, 1)$ .
- (4) Calculate the determinate of the matrix whose columns are  $u$ ,  $v$ , and  $w$  to determine if  $u$ ,  $v$ , and  $w$  are linearly independent or not, with  $u = (-2, -5, -4)$ ,  $v = (5, 4, -6)$ , and  $w = (8, 3, -4)$ .
- (5) Is the subset  $W = \{(x, y, z) \mid y \geq 0\} \subset \mathbb{R}^3$  a vector subspace of  $\mathbb{R}^3$ ? Explain why or why not.
- (6) Is the subset  $W = \{(x, y, z) \mid y = 0\} \subset \mathbb{R}^3$  a vector subspace of  $\mathbb{R}^3$ ? Explain why or why not.
- (7) Is the subset  $W = \{(x, y, z) \mid z = 1\} \subset \mathbb{R}^3$  a vector subspace of  $\mathbb{R}^3$ ? Explain why or why not.
- (8) If  $W$  is the subset of all vectors  $(x, y)$  in  $\mathbb{R}^2$  such that  $|x| = |y|$ , is  $W$  a vector subspace or not?
- (9) Suppose that  $x_0$  is a solution to the equation  $Ax = b$  (where  $A$  is a matrix and  $x$  and  $b$  are vectors). Show that  $x$  is a solution to  $Ax = b$  if and only if  $y = x - x_0$  is a solution to the system  $Ay = 0$ .
- (10) Determine whether the vectors  $v_1 = (3, 0, 1, 2)$ ,  $v_2 = (1, -1, 0, 1)$ , and  $v_3 = (4, 2, 2, 2)$  are linearly independent or dependent. If they are linearly dependent, find a non-trivial combination of them that adds up to the zero vector.
- (11) Find a basis for the subspace of  $\mathbb{R}^3$  given by  $x - 2y + 7z = 0$ .
- (12) Find a basis for the subspace of  $\mathbb{R}^3$  given by  $x = z$ .
- (13) Find a basis for the subspace of all vectors  $(x_1, x_2, x_3, x_4)$  in  $\mathbb{R}^4$  such that  $x_1 + x_2 = x_3 + x_4$ .

The following two questions are about subsets of the set of real-valued functions of the real line. We will call this set  $\mathcal{F}$ .

- (14) Is the subset of  $\mathcal{F}$  with the property that  $f(0) = 0$  a vector space?
- (15) Is the subset of  $\mathcal{F}$  with the property that  $f(-x) = -f(x)$  for all  $x$  a vector space?
- (16) Compute the Wronskian of  $f_1 = e^{-x}$ ,  $f_2 = \cos(x)$  and  $f_3 = \sin(x)$  to determine whether these three functions are linearly independent on the real line.