

Math 3280 Assignment 8, due Thursday, November 1st.

For this assignment you should read sections 5.1, 5.2, and 5.3 in the text.

- (1) Solve the initial value problem $y'' - 4y = 0$, $y(0) = 4$, $y'(0) = 2$ given that $y_1 = e^{2x}$ and $y_2 = e^{-2x}$ are both solutions to the ODE.
- (2) Find the general solution to $y'' + 6y' = 0$.
- (3) Find the general solution to $4y'' + 4y' + y = 0$.
- (4) For what second-order constant coefficient linear homogeneous ODE would $y = C_1 + C_2x$ be the general solution?
- (5) Show that the functions $3x$, $2x^2$, and $5x - 8x^2$ are linearly dependent by finding a linear combination of them that equals zero.
- (6) Find the general solution to $y'' + 10y' + 25y = 0$.
- (7) Find the general solution to $y^{(4)} - 6y^{(3)} + 9y'' = 0$.
- (8) Solve the initial value problem $y'' - 6y' + 25y = 0$, $y(0) = 6$, $y'(0) = 2$.
- (9) Find the general solution of $6y^{(4)} + 5y^{(3)} + 18y'' + 20y' - 24y = 0$ given that $y = \cos(2x)$ is a solution.
- (10) Consider the differential equation $y'' + \operatorname{sgn}(x)y = 0$, where $\operatorname{sgn}(x)$ is the sign function:

$$\operatorname{sgn}(x) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Compute the two linearly independent solutions y_1 and y_2 of this differential equation which satisfy the initial conditions $y_1(0) = 1$, $y_1'(0) = 0$ and $y_2(0) = 0$, $y_2'(0) = 1$.

- (11) Suppose that a Mr. C. M. Burns has hired you to fix his grandfather clock. The clock runs too fast, getting 5 minutes ahead per day. The time is kept by a large pendulum with a length $L = 85$ cm. The length of the pendulum is adjustable. If you use the approximation $L\theta'' + g\theta = 0$ for the angle of the pendulum, where $g = 9.8$ m/s², to what length should you adjust the pendulum so that it keeps accurate time? (Your answer should be accurate to at least three digits. You can assume that the clock rate is directly proportional to the period of the pendulum.)