Math 3280 Worksheet 10: Linear air resistance

Group members (2 to 4): ____

(1) Use the following linear air resistance model for a baseball after it is struck by a bat, separately in the x- and y-directions:

$$\frac{dv_x}{dt} = -rv_x \qquad \frac{dv_y}{dt} = -g - rv_y$$

where $v_x = \frac{dx}{dt}$ is the x-component of the velocity and $v_y = \frac{dy}{dt}$ is the y-component. The constant $g = 32$ feet/second^2. We can write the initial conditions in terms of the initial angle θ at which the ball leaves the bat, and the initial speed u - so $v_x(0) = u\cos(\theta)$ and $v_y(0) = u\sin(\theta)$. We will put $x = 0$ at home plate, and assume that the initial height is 3 feet, so $x(0) = 0$ and $y(0) = 3$.

(a) Find $x(t)$ and $y(t)$ (in units of feet and seconds) if $r = 1/5$, $\theta = \arctan(4/3)$, and $u = 135$ ft/s. (First find v_x and v_y, and then integrate with respect to t.)

(b) Extra credit. Find the minimal velocity u and optimal angle θ for a home run in center field if the field wall is 10 feet high and 350 feet from home plate, assuming the air resistance model from part (1). (An exact answer may be impossible, so numerical answers are acceptable.) You must explain your reasoning. You can hand this in separately, individually if you prefer, by October 8th.