Math 3280 Worksheet 14: Determinants

Group members (2 to 4): ______________________________

(1) Find a three by three matrix with strictly positive entries ($a_{ij} > 0$ for each entry a_{ij}) whose determinant is equal to 1. A bonus will be given for finding such a matrix with the smallest sum of entries, that is, such that $\sum_{i=1,j=1}^{i=3,j=3} a_{ij} = a_{11} + a_{12} + \ldots + a_{33}$ is as low as possible.

(2) The $n \times n$ Vandermonde determinant is defined as:

$$V(x_1, x_2, \ldots, x_n) = \det \begin{pmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \cdots & x_2^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^{n-1} \end{pmatrix}$$

Show that the 2×2 Vandermonde determinant $V(a, b) = b - a$. Then show that the 3×3 Vandermonde determinant $V(1, a, b)$ can be factored into $(a - 1)(b - 1)(b - a)$.