Math 3280 Assignment 11, due Thursday, December 4th.

This assignment covers material from section 7.4, chapter 10, and chapter 11.

(1) Compute the Laplace transform of the function

$$v(t) = \begin{cases} 1 \text{ for } t \in [0,1] \\ 0 \text{ for } t \in [-\infty, 0) \text{ and } t \in (1, \infty] \end{cases}$$

directly from the definition $\mathcal{L}(v) = \int_0^\infty e^{-st} v(t) dt$.

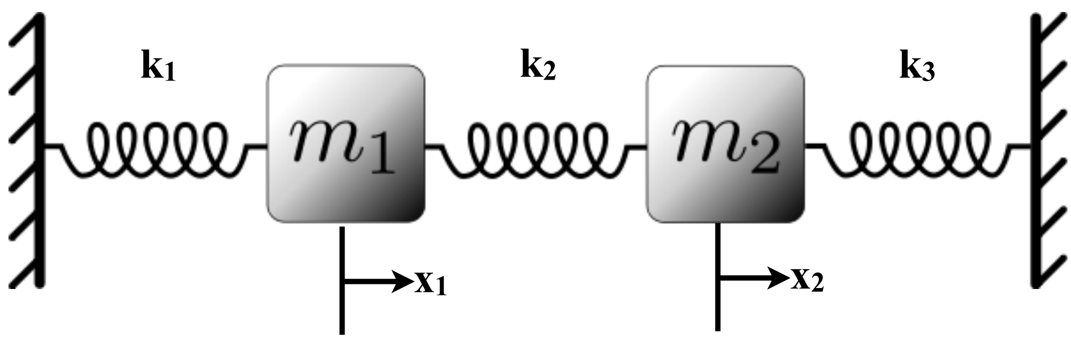
(2) Use the Laplace transform method to solve the initial value problem $x'' - x' - 2x = 0$, $x(0) = 0$, $x'(0) = 1$.

(3) Compute the Laplace transform of the sawtooth function $f(t) = t - \lfloor t \rfloor$ where $\lfloor t \rfloor$ is the *floor* function. The floor of t is the largest integer less than or equal to t. For example, $\lfloor 2.6 \rfloor = 2$.

For the next two problems, consider two blocks of mass m_1 and m_2 connected by springs to each other and to walls as shown below. The displacement of the masses from their equilibrium positions are denoted by x_1 and x_2. The stiffness of the three springs are k_1, k_2, and k_3 as shown. Compute the natural frequencies and describe the natural modes of oscillation in each of the three following cases:

(4) $k_1 = k_2 = 4$ and $k_3 = 2$, and $m_1 = 2$, $m_2 = 1$.

(5) $k_1 = k_3 = 0$ and $k_2 = 4$, and $m_1 = m_2 = 1$.



(6) Find the power series solution to the ODE $y' = 3x^2 y$ (expanded at $x = 0$). You should be able to determine each coefficient as an explicit function of its index (rather than just a recurrence relation).

(7) Show that the coefficients of the power series solution to the initial value problem $y'' - y' - y = 0$, $y(0) = 0$, $y'(0) = 1$ have the form $c_n = F_n/n!$ where F_n is the nth Fibonacci number. (The Fibonacci numbers are $1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots$, satisfying the recursion relation that each number is the sum of the previous two in the sequence.)

(8) Determine the power series solution and radius of convergence of the ODE $y'' + x^2 y = 0$.