Math 3280 Assignment 12, due Thursday December 11th.

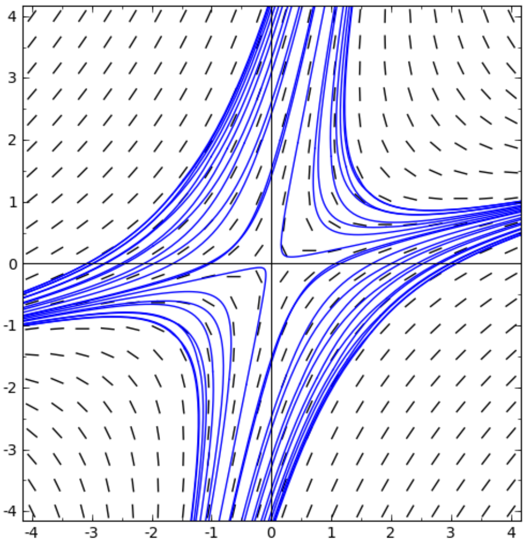
(1) Compute the equilibria of the following nonlinear differential equations, and use that information to match each equation with a trajectory plot from the following page. It may be helpful to compute the eigenvalues at an equilibrium.
  (a) $x' = x - y$, $y' = x + 3y - 4$.
  (b) $x' = 2x - y$, $y' = x - 3y$.
  (c) $x' = 2\sin(x) + \sin(y)$, $y' = \sin(x) + 2\sin(y)$.
  (d) $x' = x - 2y$, $y' = -x^3 + 4x$.
  (e) $x' = 1 - y^2$, $y' = x + 2y$.
  (f) $x' = x - 2y + 3$, $y' = x - y + 2$.

(2) Find the unique equilibrium of the system $x' = x - y$, $y' = 5x - 3y - 2$. Compute the eigenvalues of its linearization to determine the stability of the equilibrium (see Theorem 2 in section 9.2).

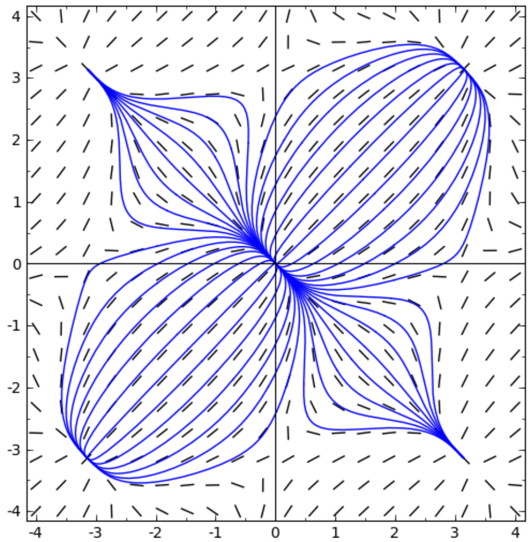
(3) In 1958, Tsuneji Rikitake formulated a simple model of the Earth's magnetic core to explain the oscillations in the polarity of the magnetic field. The equations for his model are:

$$x' = -\mu x + yz$$
$$y' = -\mu y + (z - a)x$$
$$z' = 1 - xy$$

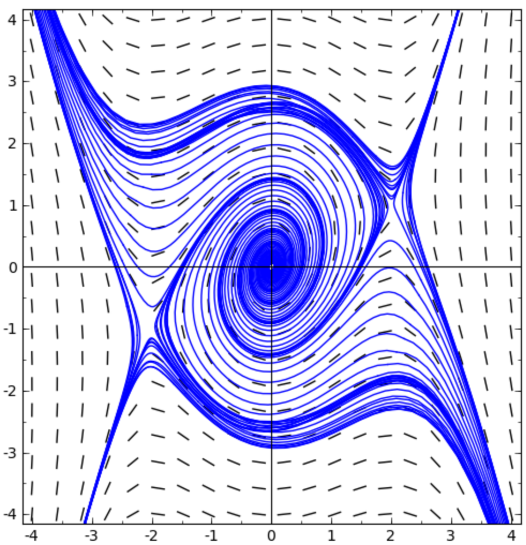
where a and μ are positive constants. Find the equilibria for this system for $a = \mu = 1$, and write down the Jacobian matrix of the linearized system at these equilibria.



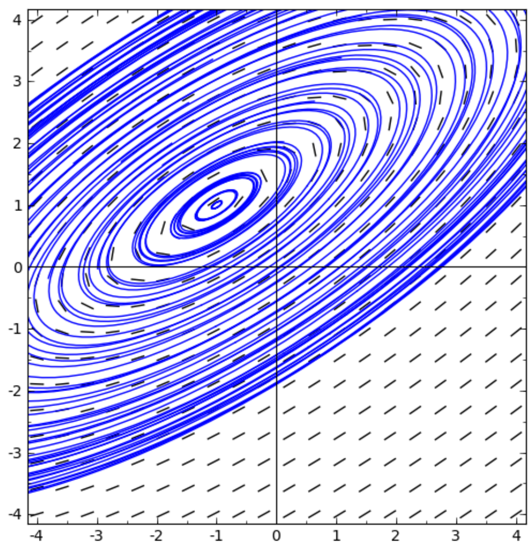
A



B



C



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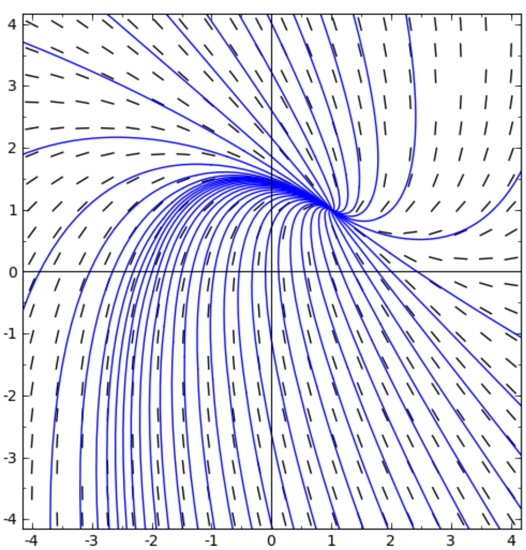
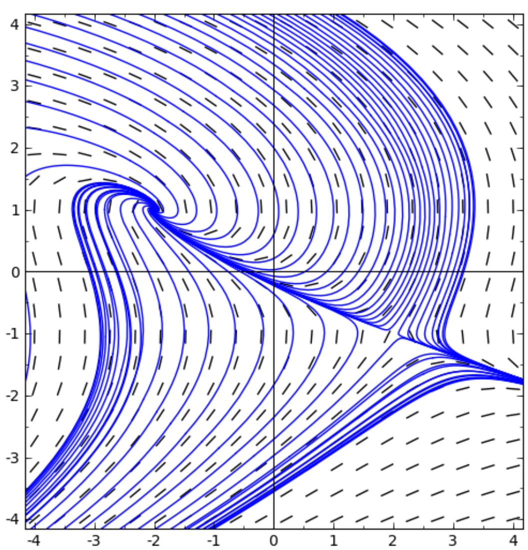


E



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