Math 3280 Assignment 11, due Friday, December 2nd.
This assignment covers material from chapter 9, and chapter 10.

(1) Compute the Laplace transform of the function

$$v(t) = \begin{cases} 1 \text{ for } t \in [0,1] \\ 0 \text{ for } t \in [-\infty, 0) \text{ and } t \in (1, \infty] \end{cases}$$

directly from the definition $\mathcal{L}(v) = \int_0^\infty e^{-st} v(t) dt$.

(2) Use the Laplace transform method to solve the initial value problem $x'' - x' - 2x = 0$, $x(0) = 0$, $x'(0) = 1$.

(3) Compute the Laplace transform of the sawtooth function $f(t) = t - \lfloor t \rfloor$ where $\lfloor t \rfloor$ is the *floor* function. The floor of t is the largest integer less than or equal to t. For example, $\lfloor 2.6 \rfloor = 2$.

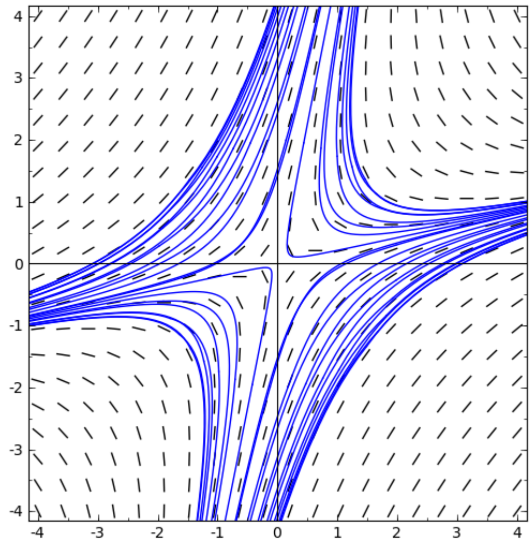
(4) Compute the equilibria of the following nonlinear differential equations, and use that information to match each equation with a trajectory plot from the following page. It may be helpful to compute the eigenvalues at an equilibrium.
   (a) $x' = x - y$, $y' = x + 3y - 4$.
   (b) $x' = 2x - y$, $y' = x - 3y$.
   (c) $x' = 2\sin(x) + \sin(y)$, $y' = \sin(x) + 2\sin(y)$.
   (d) $x' = x - 2y$, $y' = -x^3 + 4x$.
   (e) $x' = 1 - y^2$, $y' = x + 2y$.
   (f) $x' = x - 2y + 3$, $y' = x - y + 2$.

(5) Find the unique equilibrium of the system $x' = x - y$, $y' = 5x - 3y - 2$. Compute the eigenvalues of its linearization to determine the stability of the equilibrium (see Theorem 2 in section 9.2).

(6) In 1958, Tsuneji Rikitake formulated a simple model of the Earth's magnetic core to explain the oscillations in the polarity of the magnetic field. The equations for his model are:

$$x' = -\mu x + yz$$
$$y' = -\mu y + (z - a)x$$
$$z' = 1 - xy$$

where a and μ are positive constants. Find the equilibria for this system for $a = \mu = 1$, and compute the eigenvalues of the linearized system at those equilibria.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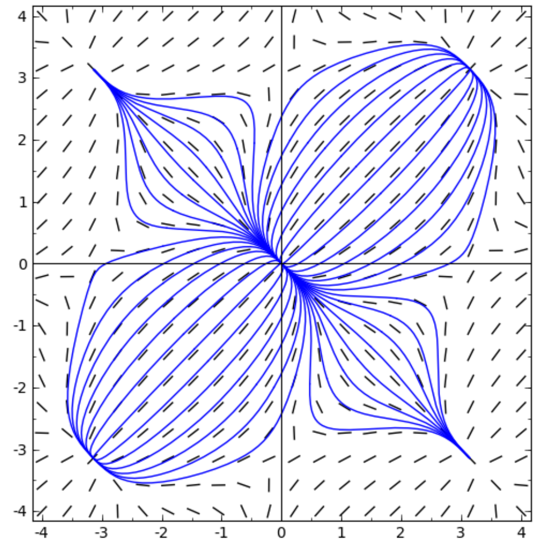
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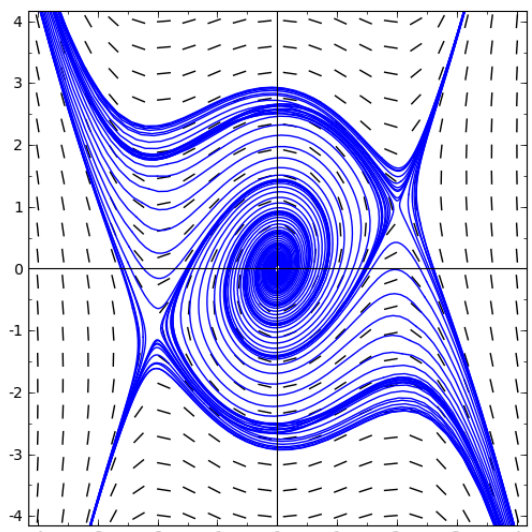
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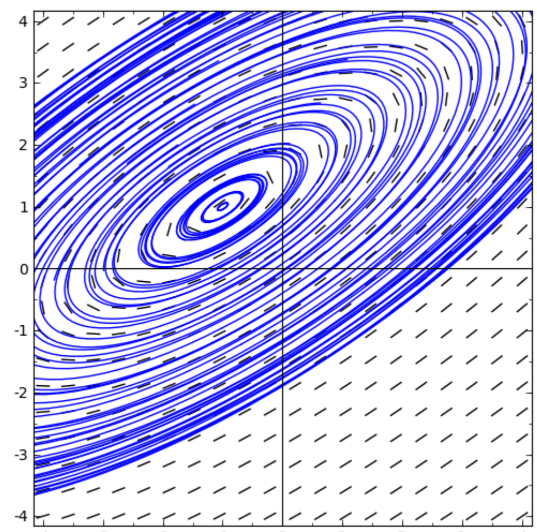
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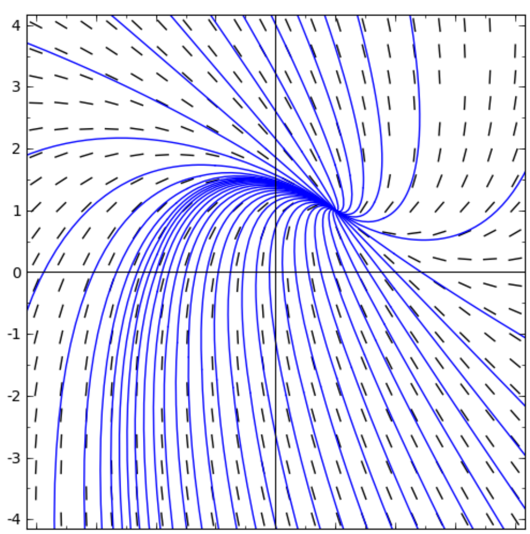
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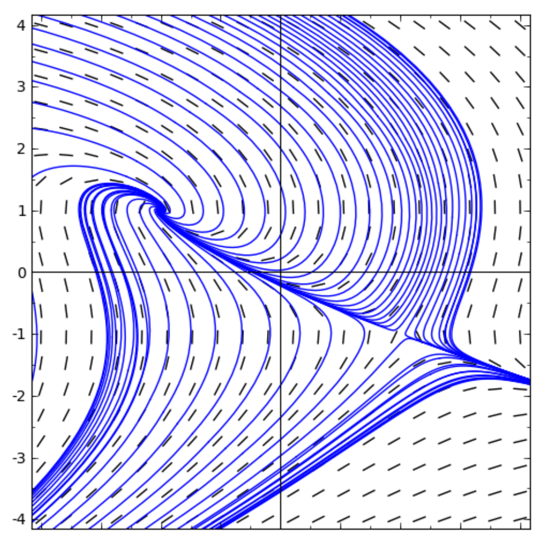
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